ECE 110 **September 22, 2008**
Professors Brunet and Loui

HOUR EXAMINATION #1

1) Write your official:
Last Name (use capital letters): Solutions (2 versions,
First Name (use capital letters):
NetId & UIN: one from each professor)

2) Write your name and section at the back of the test.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD

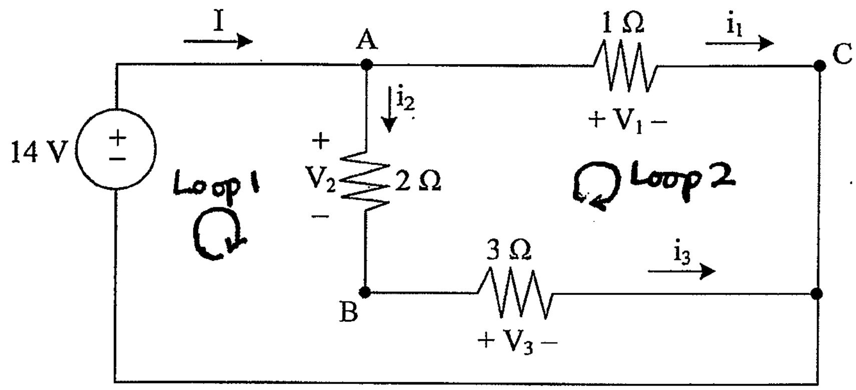
Make sure to write your name AGAIN at the top of every page of your exam.

A. Write or print clearly. Answer each problem on the exam itself. If you need extra paper, there is an extra sheet at the end of this exam. Clearly identify the problem number on any additional pages.

B. In order to receive **partial or full credit**, you must **show all your work**, e.g., your solution process, the equation(s) that you use, the values of the variables used in the equation(s), etc. You must also **include the unit of measurement** in each answer.

Students caught cheating on this exam will earn a grade of F for the entire course. Other penalties may include suspension and/or dismissal from the university.

Problem 1 *(40 points)*



a) *[6 pts.]* Write two KVL equations (use only voltage parameters). **Clearly indicate on the circuit which loop you are using.**

① $14 = V_2 + V_3$ ② $V_1 = V_2 + V_3$

b) *[6 pts.]* Write two KCL equations (use only current parameters). **Clearly indicate which node you are using.**

③ Node A: $I = i_1 + i_2$ ④ Node B: $i_2 = i_3$

c) *[6 pts.]* Write three Ohm Law equations.

⑤ $V_2 = 2 i_2$ ⑥ $V_3 = 3 i_3$ ⑦ $V_1 = 1\, i_1$

d) *[6 pts.]* Using equations above, solve the circuit. **Clearly show all steps you are using.**

By (1) and (2), $V_1 = V_2 + V_3 = 14$
By (7), $i_1 = V_1/1 = 14$
By (1), (5), (6), and (4)
$14 = 2i_2 + 3i_3 = (2+3)i_2$
so $i_2 = 14/5 = 2.8$
and $i_3 = i_2 = 2.8$

By (5), $V_2 = 2i_2 = 5.6$
By (6), $V_3 = 3i_3 = 8.4$
By (3),
$I = i_1 + i_2 = 14 + 2.8$
$= 16.8$

(write answers on next page)

Problem 1 *(continued)*

$i_1 =$	14 A	$i_2 =$	2.8 A	$i_3 =$	2.8 A
$V_1 =$	14 V	$V_2 =$	5.6 V	$V_3 =$	8.4 V
$I =$	16.8 A				

e) *[8 pts.]* Compute the SRS power for each component. **Show work.**

The SRS power at the 14V source is $14 \times (-I)$
$= -14 \times 16.8 = -235.2$

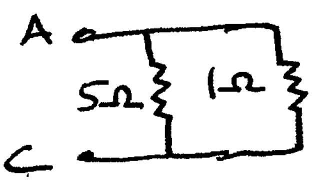
$P(1\Omega) = V_1 \times i_1 = 14 \times 14 = 196$
$P(2\Omega) = V_2 \times i_2 = 5.6 \times 2.8 = 15.68$
$P(3\Omega) = V_3 \times i_3 = 8.4 \times 2.8 = 23.52$

Notice that the total power is zero.

P (14 V) =	−235.2 W	P (2 Ω) =	15.68
P (1 Ω) =	196 W	P (3 Ω) =	23.52 W

f) *[8 pts.]* Compute i_3 using the Current Divider Rule. **Clearly show work** (showing a formula is not enough — explain what you do).

Since the 2Ω and 3Ω resistors are in series, the resistance between A and C is equivalent to



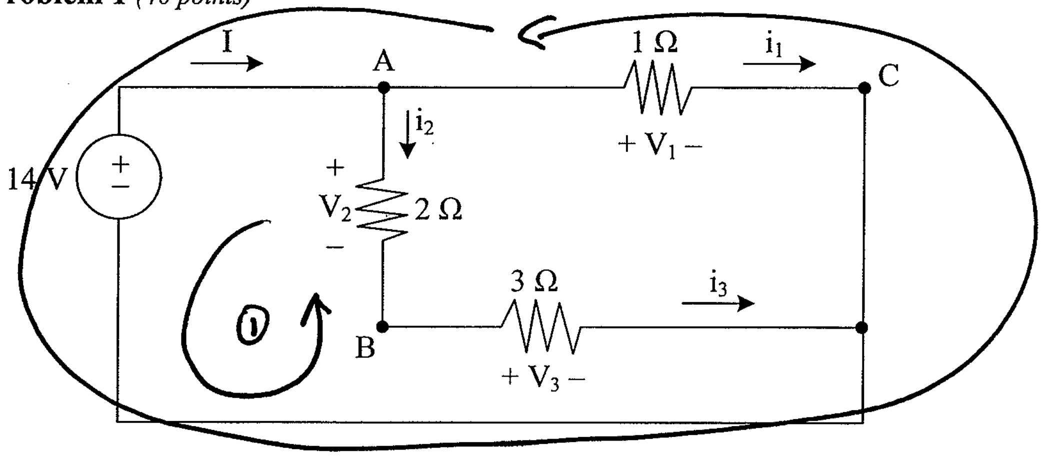
Now the 5Ω and 1Ω resistances are in parallel, so the equivalent resistance $R_{AC} = \frac{5 \times 1}{5+1} = \frac{5}{6}\Omega$.

Now by CDR, $i_3 = \frac{R_{AC}}{5} \times I = \frac{5/6}{5} \times 16.8 = 2.8A$

$i_3 =$ 2.8 A

M.-C. Brunet

Problem 1 *(40 points)*



a) *[6 pts.]* Write two KVL equations (use only voltage parameters). **Clearly indicate on the circuit which loop you are using.**

① $14 = V_2 + V_3$ ② $14 = V_1$

b) *[6 pts.]* Write two KCL equations (use only current parameters). **Clearly indicate which node you are using.**

③ @A : $I = i_1 + i_2$ ④ @B : $i_2 = i_3$

c) *[6 pts.]* Write three Ohm Law equations.

⑤ $V_1 = 1 \cdot i_1$ ⑥ $V_2 = 2 \cdot i_2$ ⑦ $V_3 = 3 \cdot i_3$

d) *[6 pts.]* Using equations above, solve the circuit. **Clearly show all steps you are using.**

- clearly ② states that $V_1 = 14$ volt ; using ⑤ $i_1 = V_1/1 \Rightarrow i_1 = 14A$
- use ④ in ⑦ $\Rightarrow V_3 = 3 \cdot i_2$; ⑥ $\Rightarrow i_2 = V_2/2$ $\Rightarrow V_3 = 3/2\, V_2$; use this in ①: $14 = V_2 + 3/2\, V_2 = 5/2\, V_2 \Rightarrow V_2 = 28/5 = 5.6V$
- use ⑥ : $i_2 = V_2/2 = 5.6/2 \Rightarrow i_2 = 2.8A \Rightarrow$ ④ $i_3 = 2.8A$
- use ⑦ : $V_3 = 3 \cdot i_3 = 3 \times 2.8 \Rightarrow V_3 = 8.4$ volt (we could have used ① too to find V_3)
- use ③: $I = i_1 + i_2 = 14 + 2.8 \Rightarrow I = 16.8A$

(write answers on next page)

M.-C. Brunet

Problem 1 *(continued)*

$i_1 = \boxed{14\,A}$ $i_2 = \boxed{2.8A}$ $i_3 = \boxed{2.8A}$

$V_1 = \boxed{14 \text{ volts}}$ $V_2 = \boxed{5.6 \text{ Volt}}$ $V_3 = \boxed{8.4 \text{ volt}}$

$I = \boxed{16.8\,A}$

e) *[8 pts.]* Compute the SRS power for each component. **Show work.**

For the 14volt source, we need to take $(-I)$ to have $(14V, -I)$ in SRS

$\Rightarrow P(14\text{volts}) = 14 \times (-16.8) = -235.2\,W.$

$P(1\Omega) = V_1 . i_1 = 14 \times 14 = 196W$ $P(2\Omega) = V_2 . i_2 = 5.6 \times 2.8 = 15.68W$

$P(3\Omega) = V_3 . i_3 = 8.4 \times 2.8 = 23.52W$

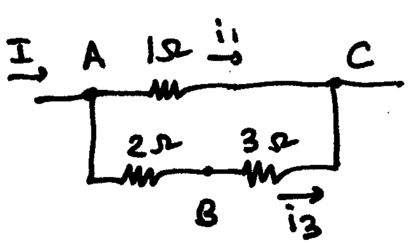
notice that Σ Powers $= -235.2 + 196 + 15.68 + 23.52 = 0$!!

$P(14\text{ V}) = \boxed{-235.2W}$ $P(2\,\Omega) = \boxed{15.68\,W}$

$P(1\,\Omega) = \boxed{196\,W}$ $P(3\,\Omega) = \boxed{23.52W}$

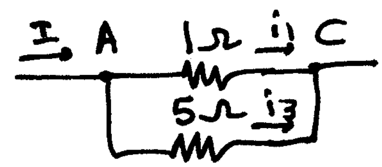
f) *[8 pts.]* Compute i_3 using the Current Divider Rule. **Clearly show work** (showing a formula is not enough — explain what you do).

current divider rule applies for resistors in parallel.



is the same as:

(2 and 3 are in serie)



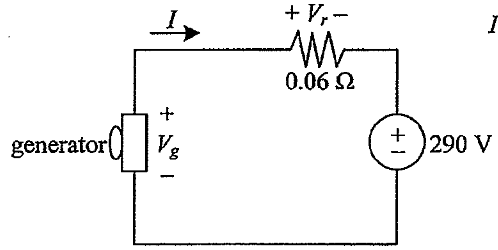
now we have a system of 2 resistors in parallel (1Ω and 5Ω) with current I entering the system, and i_3 the current thru the 5Ω.

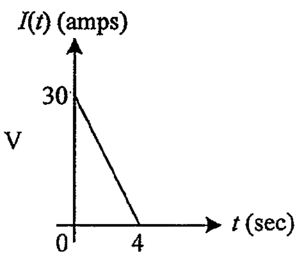
$i_3 = \frac{Req}{5} \times I$ $Req = \frac{5 \times 1}{5+1} = \frac{5}{6}$

$\Rightarrow i_3 = \frac{5/6}{5} \times 16.8 = \frac{16.8}{6} = 2.8A$

$i_3 = \boxed{2.8\,A}$

Problem 2 *(20 points)* The kinetic energy of a Toyota Prius car is 53 kilojoules (kJ) when it travels at 20 miles per hour (mph). The car's regenerative braking system has a generator that converts the kinetic energy into electricity, which recharges the battery. We model the battery as a 290 V ideal voltage source and a 0.06 Ω resistor. Starting from a speed of 20 mph, the brakes are applied at time $t = 0$, and the current $I(t)$ from the generator decreases linearly from 30 A to 0 A at time $t = 4$ seconds (sec).





a) *[2 pts.]* Write a KVL equation that relates V_r and V_g.

$V_g = V_r + 290$

b) *[4 pts.]* Determine the value of V_g when I reaches 0. **Explain your reasoning.**

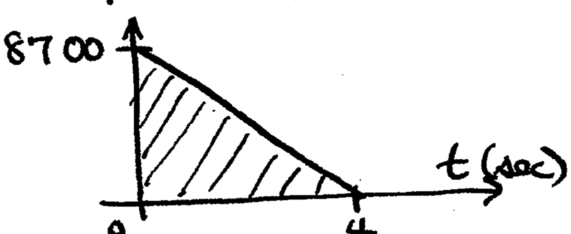
By Ohm's Law, when $I=0$, $V_r = 0.06 \times I = 0$.
So by part (a), $V_g = V_r + 290 = 0 + 290$

$V_g =$ $\boxed{290\ V}$

c) *[8 pts.]* Determine the average SRS power P_{avg} for the 290 V voltage source between 0 and 4 seconds. **Show your work.**

$290\ V \times 30A = 8700\ W$ initial power

Power at voltage source
P(t) (kW)



$P_{avg} = \dfrac{\text{area under curve}}{4} = \frac{1}{4}\left(\frac{1}{2} \times 4 \times 8700\right)$
$= 4350$

$P_{avg} =$ $\boxed{4350\ W}$

d) *[6 pts.]* Determine the total amount of energy returned to the voltage source and the efficiency of the regenerating system in this case.

Energy $= P_{avg} \times 4\,sec = 4350\ W \times 4\ sec$
$= 17.4\ kJ$

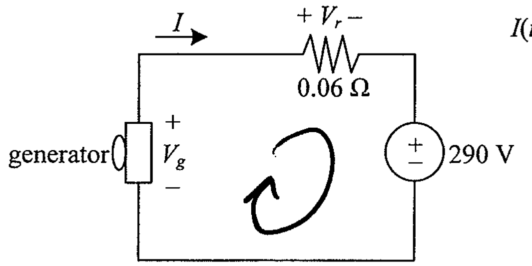
Efficiency $= \dfrac{17.4\ kJ}{53\ kJ} = 0.328$

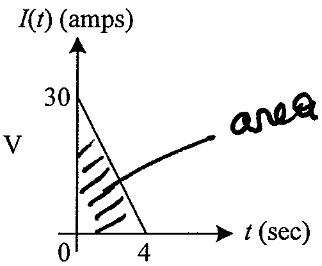
Energy = $\boxed{17.4\ kJ}$

Efficiency = $\boxed{32.8\ \%}$

M.-C. Brunet

Problem 2 *(20 points)* The kinetic energy of a Toyota Prius car is 53 kilojoules (kJ) when it travels at 20 miles per hour (mph). The car's regenerative braking system has a generator that converts the kinetic energy into electricity, which recharges the battery. We model the battery as a 290 V ideal voltage source and a 0.06 Ω resistor. Starting from a speed of 20 mph, the brakes are applied at time $t = 0$, and the current $I(t)$ from the generator decreases linearly from 30 A to 0 A at time $t = 4$ seconds (sec).





$\text{area} = \frac{30 \times 4}{2} = 60$

a) *[2 pts.]* Write a KVL equation that relates V_r and V_g.

$V_g = V_r + 290$

b) *[4 pts.]* Determine the value of V_g when I reaches 0. **Explain your reasoning.**

when I reaches 0, $V_r = I \times 0.06$ reaches 0 too

from (a) $V_g = V_r + 290$ reaches 290

$V_g = \boxed{290 \text{ volt}}$

c) *[8 pts.]* Determine the average SRS power P_{avg} for the 290 V voltage source between 0 and 4 seconds. **Show your work.**

$P_{avg}(290\text{V}) = \text{average}(290 \times I) = \underbrace{290}_{\text{constant}} \times \text{average}(I)$

$\text{average}(I) = \frac{\text{area under } I}{\text{period}} = \frac{60}{4}$ (see work close to graph)

$\Rightarrow \text{average}(I) = 15\text{A}$ $\quad P_{avg} = 290 \times 15$

$P_{avg} = \boxed{4350 \text{ W}}$

d) *[6 pts.]* Determine the total amount of energy returned to the voltage source and the efficiency of the regenerating system in this case.

$\text{Energy}(290\text{V}) = P_{avg}(290\text{V}) \times 4 = 4350 \times 4 = 17{,}400 \text{ J}$

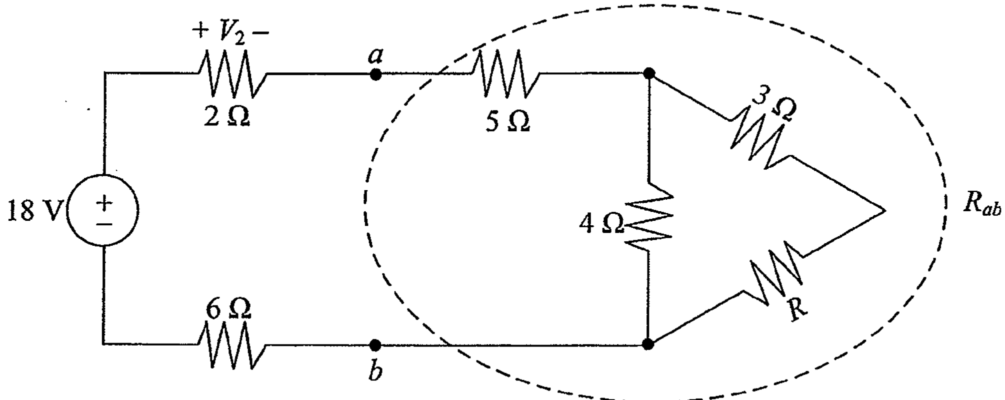
$\text{Efficiency} = \frac{17{,}400}{53{,}000} \times 100 = 32.8\%$

Energy = $\boxed{17{,}400 \text{ J}}$

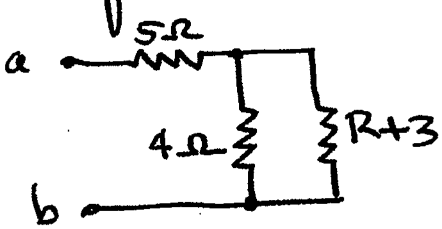
Efficiency = $\boxed{32.8\%}$

Problem 3 *(20 points)*

a) *[10 pts.]* Show that the equivalent resistance R_{ab} between node a and node b is $\frac{9R+47}{R+7}\Omega$.
Explain your reasoning using diagrams, formulas, **and words**.



R and 3 Ω are in series.
Equivalent to



Now 4 Ω and R+3 in parallel
Equivalent to

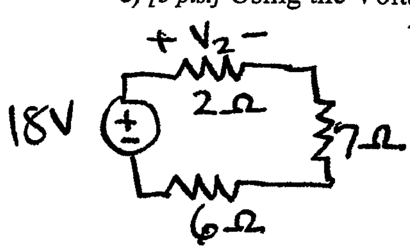
$$R_{ab} = 5 + \frac{4(R+3)}{R+7} = \frac{5(R+7)+4(R+3)}{R+7} = \frac{5R+35+4R+12}{R+7}$$

$$= \frac{9R+47}{R+7}\ \Omega$$

b) *[5 pts.]* Using part a), explain why it is **impossible** to choose R so that $R_{ab} < 6\ \Omega$.

By part (a), $R_{ab} < 6$ requires $\frac{9R+47}{R+7} < 6$, hence
$9R+47 < 6(R+7) = 6R+42$, or $3R < -5$ and $R < -5/3 < 0$.
But that's impossible since a resistance R must be positive.

c) *[5 pts.]* Using the Voltage Divider Rule, determine the value of V_2 when $R_{ab} = 7\ \Omega$.



The three resistors in series are equivalent to $2+7+6 = 15\Omega$.

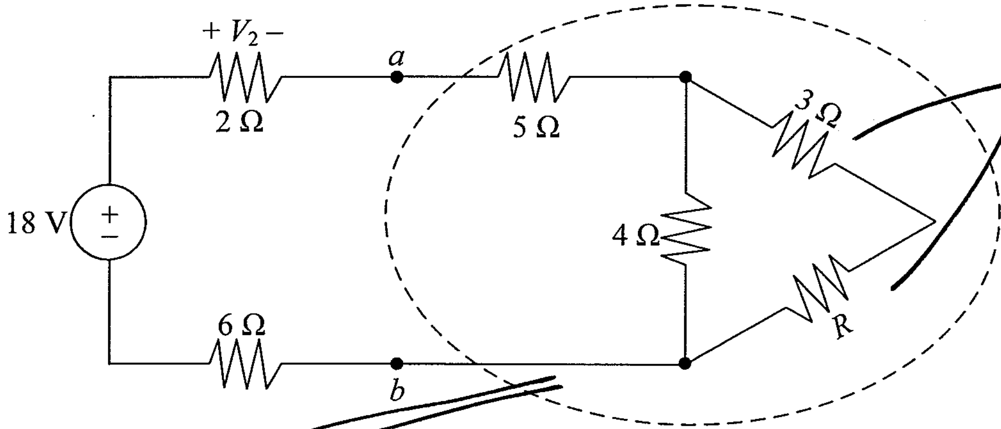
By VDR, $V_2 = \frac{2}{15}\times 18 = 2.4$ $V_2 = \boxed{2.4\ V}$

M-C. Brunet

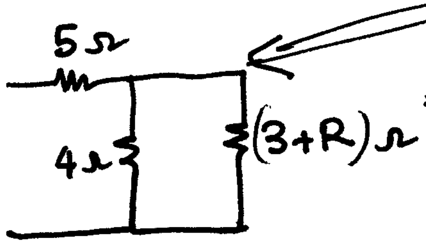
Problem 3 *(20 points)*

a) *[10 pts.]* Show that the equivalent resistance R_{ab} between node a and node b is $\dfrac{9R+47}{R+7}\,\Omega$.

Explain your reasoning using diagrams, formulas, **and words**.

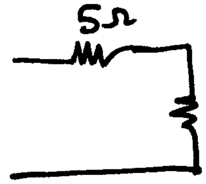


are in series



⇒ now 4 Ω and (3+R) Ω are in //

⇒



$$\frac{4(3+R)}{4+3+R} = \frac{12+4R}{7+R}$$

now 5 and $\dfrac{12+4R}{7+R}$ are in serie

$$\Rightarrow R_{ab} = 5 + \frac{12+4R}{7+R} = \frac{5(7+R)+12+4R}{7+R}$$

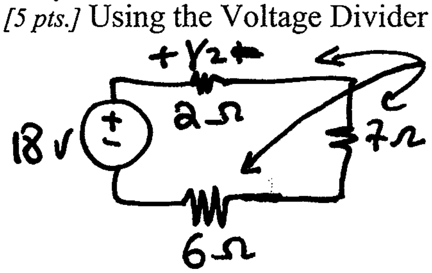
$$= \frac{35+5R+12+4R}{7+R} = \frac{47+9R}{7+R}\,\Omega \quad \text{proved.}$$

b) *[5 pts.]* Using part a), explain why it is **impossible** to choose R so that $R_{ab} < 6\ \Omega$.

if $R_{ab} < 6 \Rightarrow \dfrac{47+9R}{7+R} < 6 \Rightarrow 47+9R < 42+6R \Rightarrow 6R < -5$

which would imply that $R < 0$ (which is not possible) -

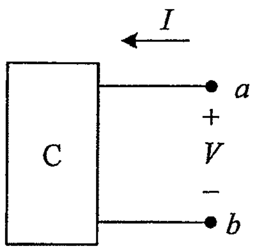
c) *[5 pts.]* Using the Voltage Divider Rule, determine the value of V_2 when $R_{ab} = 7\ \Omega$.



$$V_2 = \frac{2}{2+7+6} \times 18$$

$V_2 =$ $\boxed{2.4 \text{ volts}}$

Problem 4 *(20 points)* The component C below has the I-V characteristic shown, with an unknown constant s. The units are amps and volts.

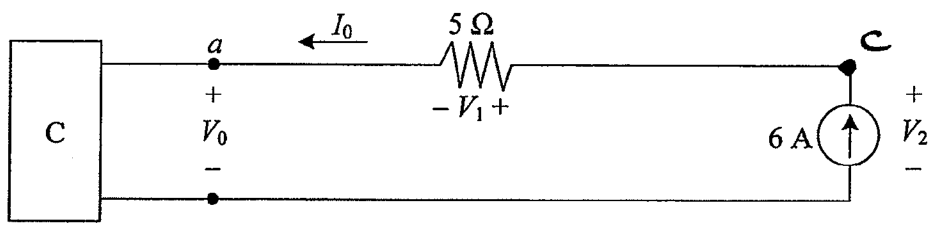


$$I = s\,V - 4.8$$

a) *[5 pts.]* Can C be a single resistor (with no other components)? Answer *yes* or *no* or *insufficient information*, and <u>justify your answer</u>.

No. For a resistor R, when $V=0$, $I=\frac{V}{R}=0$ by Ohm's Law. But with the I-V characteristic for C, when $V=0$, $I=-4.8$. Thus C can not be a resistor.

b) When C is connected to the resistor and ideal current source shown below, the operating voltage V_0 is 9 V.



(i) *[5 pts.]* Determine the value of s (in amps/volts). **Show work.**

By KCL at c, $I_0 = 6$. From the I-V characteristic with $V_0 = 9$, we have $6 = 9s - 4.8$

$$s = \frac{10.8}{9} = 1.2$$

$s =$ $\boxed{1.2\ A/V}$

(ii) *[6 pts.]* Determine the values of V_1 and V_2. **Show work.**

By KVL, $V_0 + V_1 = V_2$

By Ohm's Law, $V_1 = 5I_0 = 5\times 6 = 30$

So $V_2 = V_0 + V_1 = 30 + 9 = 39$

$V_1 =$ $\boxed{30\ V}$

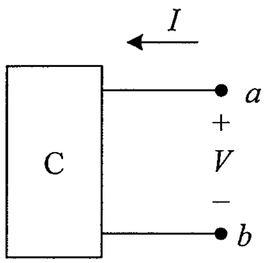
$V_2 =$ $\boxed{39\ V}$

(iii) *[4 pts.]* Does the 6 A current source generate or dissipate electrical power? <u>**Justify your answer**</u>. [HINT: $V_2 > 0$]

The SRS power at the 6A source is $-6\times V_2 = -234\,W < 0$, so it is <u>generating</u> electrical power.

M.-C. Brunet

Problem 4 *(20 points)* The component C below has the I-V characteristic shown, with an unknown constant s. The units are amps and volts.

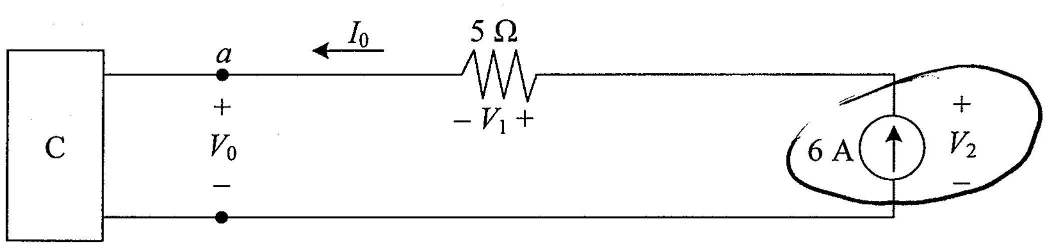


$I = s\,V - 4.8$ (*)

a) *[5 pts.]* Can C be a single resistor (with no other components)? Answer *yes* or *no* or *insufficient information*, and **justify your answer**.

no. for a single resistor $I = \frac{1}{R} V$ when $V=0$ $I=0$ -
for the given case : when $V=0$, $I = -4.8$ (not zero)

b) When C is connected to the resistor and ideal current source shown below, the operating voltage V_0 is 9 V.



not in SRS

(i) *[5 pts.]* Determine the value of s (in amps/volts). **Show work.**

By KCL : $I_0 = 6A$ $V_0 = 9V$ given - (V_0, I_0) satisfie the given IV ch(*) $6 = s \times 9 - 4.8$
$\Rightarrow s = (6+4.8)/9 = 1.2$

$s = \boxed{1.2 \text{ A/volt}}$

(ii) *[6 pts.]* Determine the values of V_1 and V_2. **Show work.**

- since $I_0 = 6A$: $V_1 = I_0 . 5 = 6 \times 5 = 30$ volt
- By KVL : $V_2 = V_1 + V_0 = 30 + 9$

$V_1 = \boxed{30 \text{ volt}}$

$V_2 = \boxed{39 \text{ volt}}$

(iii) *[4 pts.]* Does the 6 A current source generate or dissipate electrical power? **Justify your answer**. [HINT: $V_2 > 0$]

The power in SRS is $V_2 \times (-6)$; $V_2 > 0 \Rightarrow$ power < 0
so it is generating electrical power -