ECE 110
Professors Brunet and Loui

September 21, 2009

HOUR EXAMINATION #1

Write your official (*not a nickname*):

Last Name (use capital letters): Solutions (2 versions,
First Name (use capital letters): one from each professor)
NetId & UIN:________________________________

2) Write your name and section at the back of the test.

<u>DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD</u>

<u>Make sure to write your name AGAIN at the top of every page of your exam.</u>

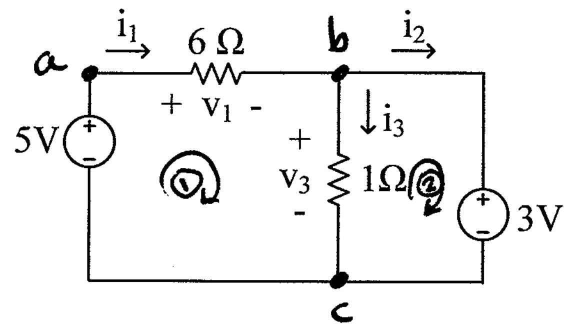
A. Write or print clearly. Answer each problem on the exam itself. If you need extra paper, there is an extra sheet at the end of this exam. Clearly identify the problem number on any additional pages.

B. In order to receive **partial or full credit**, you must **show all your work**, e.g., your solution process, the equation(s) that you use, the values of the variables used in the equation(s), etc. You must also **include the unit of measurement** in each answer.

Students caught cheating on this exam will earn a grade of F for the entire course. Other penalties may include suspension and/or dismissal from the university.

Loui

Problem 1 (15 *points*)



(a) (*2 pts.*) Count the number of nodes in the circuit above and label them on the diagram:

Labels a, b, c.

number of nodes $\boxed{3}$

(b) (*3 pts.*) Count the number of unknowns in the circuit above and name them:

number of unknowns $\boxed{5}$

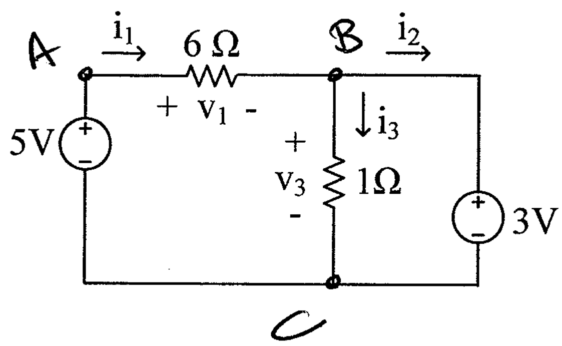
unknowns $\boxed{i_1, i_2, i_3, V_1, V_3}$

(c) (*10 pts.*) Write as many basic circuit equations as necessary to solve the circuit. No need to solve!

Ohm's law	Kirchhoff current law (use currents only! no voltages)	Kirchhoff voltage law (use voltages only! no currents)
$V_1 = 6 i_1$ $V_3 = 1 \cdot i_3$	At b: $i_1 = i_2 + i_3$	Loop ①: $V_1 + V_3 = 5$ Loop ②: $V_3 = 3$

Brunet.

Problem 1 (15 *points*)



(a) (*2 pts.*) Count the number of nodes in the circuit above and label them on the diagram:

number of nodes: 3

(b) (*3 pts.*) Count the number of unknowns in the circuit above and name them:

number of unknowns: 5

unknowns: i_1, v_1, i_2, i_3, v_3

(c) (*10 pts.*) Write as many basic circuit equations as necessary to solve the circuit. No need to solve!

need 5 equations (5 unknowns) –

Ohm's law	Kirchhoff current law (use currents only! no voltages)	Kirchhoff voltage law (use voltages only! no currents)
2 resistors.	@ node B:	left loop:
$\frac{v_1}{i_1} = 6$	$i_1 = i_2 + i_3$	$-5 + v_1 + v_3 = 0$
$\frac{v_3}{i_3} = 1$		right loop:
		$-v_3 + 3 = 0$

Loui

Problem 2 (20 *points*)

For each statement, check the one best answer.

(a) The I-V characteristic of a circuit that contains only ideal current and voltage sources and resistors is
☑ always linear ☐ sometimes linear ☐ never linear ☐ a sine wave

(b) The magnitude of the current through an ideal voltmeter is
☐ infinite ☐ very large ☐ very small ☑ zero

(c) When the ground is changed from one node to another node in the same circuit
☐ every current increases or decrease ☐ the node with the lowest voltage changes
☑ every current remains the same as before ☐ the node with the highest voltage changes

(d) When computed in the standard reference system, the power for a resistor is
☐ strictly negative ☐ negative or zero ☑ positive or zero ☐ strictly positive
when current is zero

(e) When computed in the same reference system, the sum of the powers of all circuit elements is
☐ always positive ☑ always zero ☐ always negative ☐ none of these

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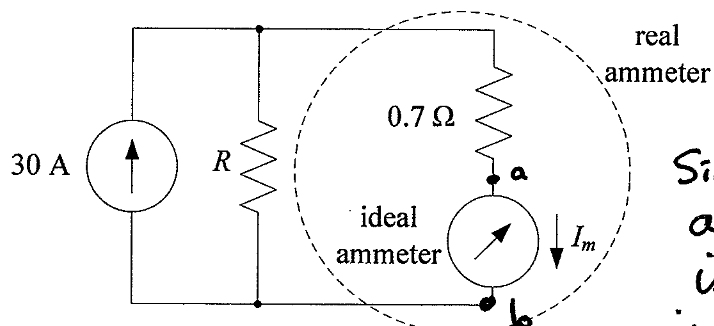
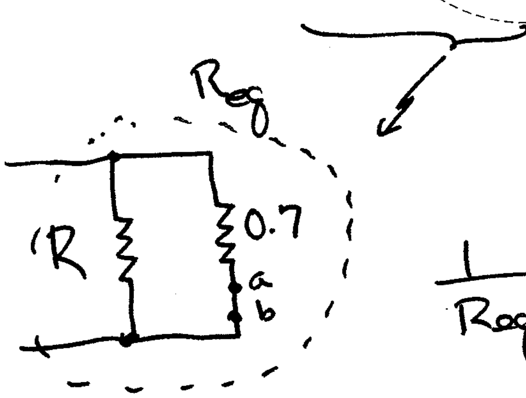
Problem 3 *(10 points)*

We model a real ammeter by an ideal ammeter in series with a 0.7 Ω resistor. We wish to use the real ammeter for current values as large as 30 amps by attaching a resistor R in parallel with the real ammeter, and scaling the reading. Use the Current Divider Rule to determine the maximum value of R so that the current through the ammeter remains less than 6 amps, i.e., $I_m < 6$.



$R =$ $\boxed{0.175\ \Omega}$

Since the voltage drop V_{ab} across the ideal ammeter is zero, the ideal ammeter is effectively equivalent to a short. Resistors R and 0.7 Ω are essentially in parallel, so



$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{0.7} = \frac{0.7+R}{0.7R}, \quad R_{eq} = \frac{0.7R}{0.7+R}$$

By CDR, $I_m = 30 \times \frac{R_{eq}}{0.7} = \frac{30R}{0.7+R} < 6$ when

$30R < 6(0.7+R)$, $5R < 0.7+R$, $4R < 0.7$, $R < 0.175\,\Omega$

Brunet

Problem 2 (20 *points*)

For each statement, check the one best answer.

(a) The I-V characteristic of a circuit that contains only ideal current and voltage sources and resistors is
- ☒ always linear
- ☐ sometimes linear
- ☐ never linear
- ☐ a sine wave

(b) The magnitude of the current through an ideal voltmeter is
- ☐ infinite
- ☐ very large
- ☐ very small
- ☒ zero

(c) When the ground is changed from one node to another node in the same circuit
- ☐ every current increases or decrease
- ☒ every current remains the same as before
- ☐ the node with the lowest voltage changes
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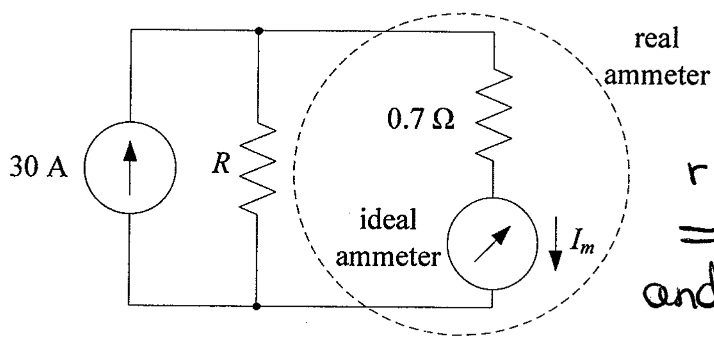
- ☐ strictly negative
- ☐ negative or zero
- ☒ positive or zero
- ☐ strictly positive

(e) When computed in the same reference system, the sum of the powers of all circuit elements is
- ☐ always positive
- ☒ always zero
- ☐ always negative
- ☐ none of these

Problem 3 *(10 points)*

We model a real ammeter by an ideal ammeter in series with a 0.7 Ω resistor. We wish to use the real ammeter for current values as large as 30 amps by attaching a resistor *R* in parallel with the real ammeter, and scaling the reading. Use the Current Divider Rule to determine the maximum value of *R* so that the current through the ammeter remains less than 6 amps, i.e., $I_m < 6$.



$R = \boxed{0.175\Omega}$

r=0 for ideal ammeter
⟹ R and 0.7 are in parallel
and we can apply the CDR.

$$\text{i.e. } I_m = \frac{R_{eq}}{0.7} \times 30 = \frac{(0.7 \times R)/(0.7+R)}{0.7} \times 30 = \frac{R}{0.7+R} \times 30$$

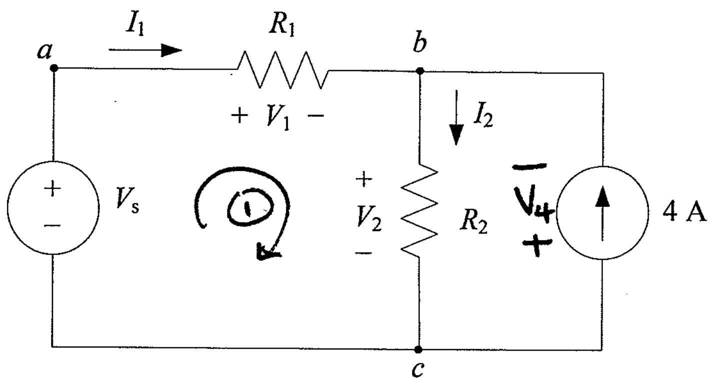
$$I_m < 6 \Rightarrow \frac{R}{0.7+R} \times 30 < 6 \Rightarrow 30R < 4.2 + 6R$$

$$\Rightarrow 24R < 4.2 \Rightarrow R < \frac{4.2}{24} \Rightarrow R < 0.175$$

Loui

Problem 4 *(18 points)*

In this circuit, $I_1 = 8$ A and $V_2 = 9$ V, but V_s is an unknown constant.



$I_1 = 8$ A

$V_2 = 9$ V

(a) *(3 pts.)* Use KCL at node *b* to determine the value of the current I_2.

By KCL at b, $I_2 = I_1 + 4 = 8 + 4 = 12A$

$I_2 = \boxed{12\text{ A}}$

(b) *(3 pts.)* Use Ohm's Law to determine the value of the resistance R_2.

By Ohm's Law, $R_2 = \frac{V_2}{I_2} = \frac{9}{12} = 0.75\Omega$

$R_2 = \boxed{0.75\Omega}$

(c) *(4 pts.)* Does the 4 A current source generate or dissipate electrical energy? **Explain your reasoning.**

Since $V_4 = -V_2 = -9V$, the SRS power at the current source is $-9V \times 4A = -36W < 0$. Since the SRS power is negative, the current source is generating electrical energy.

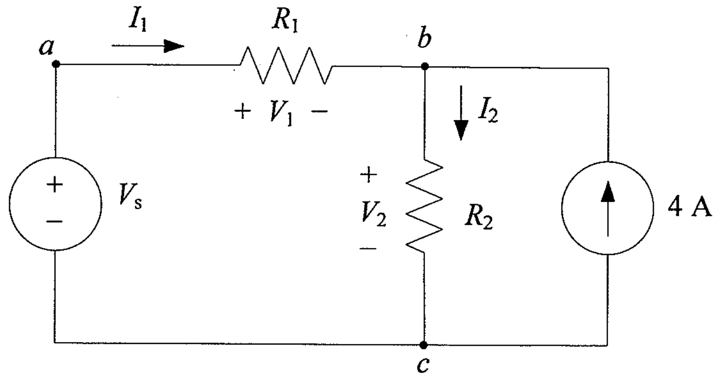
(d) *(8 pts)* Prove that $V_s > V_2$, with formulas **and words.** (Hint: Start with a KVL equation.)

By KVL in loop ①, $V_1 = V_s - V_2$. By Ohm's Law in SRS, $V_1 = I_1 R_1 = 8R_1 > 0$ because a resistance is always positive. Therefore, $V_s - V_2 = V_1 > 0$, hence $V_s > V_2$.

Brunet

Problem 4 *(18 points)*

In this circuit, $I_1 = 8$ A and $V_2 = 9$ V, but V_s is an unknown constant.



$I_1 = 8$ A

$V_2 = 9$ V

(a) (*3 pts.*) Use KCL at node *b* to determine the value of the current I_2.

$$\underbrace{I_1 + 4}_{\text{enter}} = \underbrace{I_2}_{\text{leave}} \Rightarrow I_2 = 8+4$$

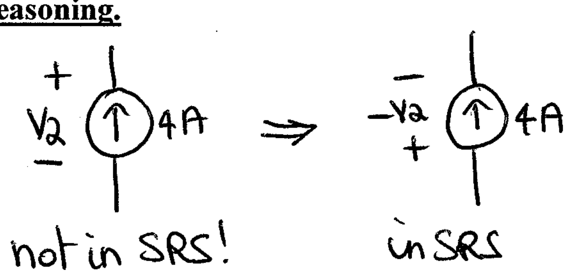
$I_2 = \boxed{12\text{ A}}$

(b) *(3 pts.)* Use Ohm's Law to determine the value of the resistance R_2.

$$R_2 = \frac{V_2}{I_2} = \frac{9}{12}$$

$R_2 = \boxed{0.75\ \Omega}$

(c) *(4 pts.)* Does the 4 A current source generate or dissipate electrical energy? **Explain your reasoning.**



$P_{SRS} = -V_2 \times 4 = -9 \times 4$

it is negative

$\Rightarrow$ it generates energy

(d) *(8 pts)* Prove that $V_s > V_2$, with formulas **and words.** (Hint: Start with a KVL equation.)

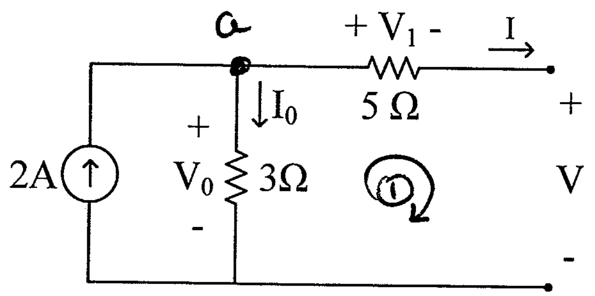
$-V_s + V_1 + V_2 = 0$ (KVL left loop)

$\Rightarrow V_s = V_1 + V_2$ if $V_1 > 0$ then surely $V_s > V_2$.

$V_1 = R_1 I_1$ $\quad$ $I_1 > 0$ $(I_1 = 8A)$, $R_1 > 0$ (resistor) $\Rightarrow V_1 > 0$!

Loui

Problem 5 (*15 points*)



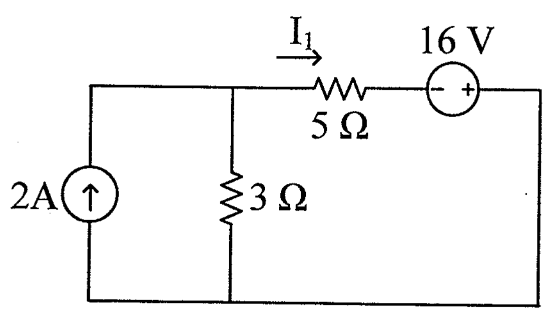
Show work for all parts.
hint: parts a and b are independent (i.e. can be done in any order)

The units are amps and volts.

(a) (*9 pts.*) Prove that $I = -\frac{1}{8}V + \frac{6}{8}$ (use <u>any</u> method of your choice). Partial credit will be given for correct partial work.

By KCL at node a, $2 = I_0 + I$.
By Ohm's Law, $V_1 = 5I$ and $V_0 = 3I_0$
By KVL in loop ①, $V_0 = V_1 + V$.
So $3I_0 = V_0 = V_1 + V = 5I + V$ and $3I_0 = 3(2 - I) = 6 - 3I$.
Consequently $5I + V = 6 - 3I$
$8I = 6 - V$
$I = -\frac{1}{8}V + \frac{6}{8}$

(b) (*6 pts.*) Find the numerical value of I_1.

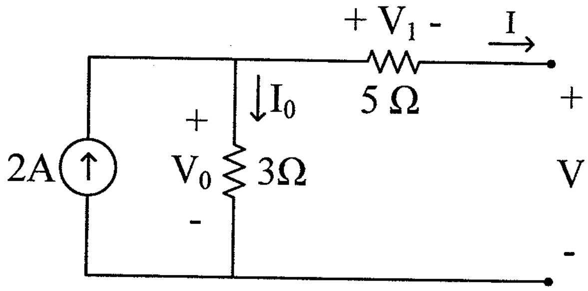


According to the I-V characteristic of part (a),
$I = I_1 = -\frac{1}{8}(-16) + \frac{6}{8}$
$= 2 + \frac{6}{8} = 2.75A$

$I_1 = \boxed{2.75A}$

Brunet

Problem 5 (*15 points*)



Show work for all parts.
hint: parts a and b are independent (i.e. can be done in any order)

The units are amps and volts.

(a) (*9 pts.*) Prove that $I = -\frac{1}{8}V + \frac{6}{8}$ (use <u>any</u> method of your choice). Partial credit will be given for correct partial work.

Ohm $\frac{V_0}{I_0} = 3$ ① $\frac{V_1}{I} = 5$ ②

KCL $2 = I_0 + I$ ③

KVL $-V_0 + V_1 + V = 0$ ④

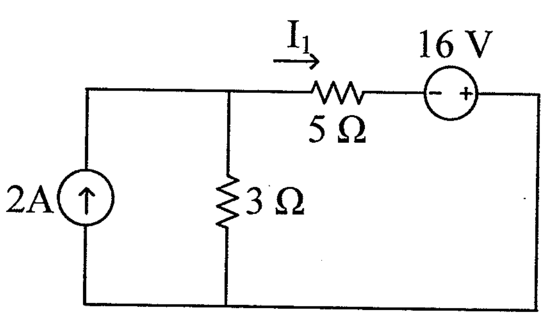
$$② \Rightarrow I = \frac{V_1}{5} \overset{④}{=} \frac{V_0 - V}{5} \overset{①}{=} \frac{3I_0 - V}{5} \overset{③}{=} \frac{3(2-I) - V}{5}$$

$$\Rightarrow 5I = 6 - 3I - V$$

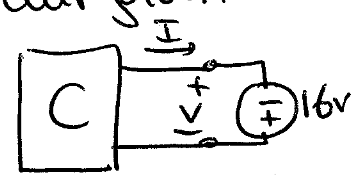
$$\Rightarrow 8I = 6 - V$$

$$\Rightarrow I = -\frac{V}{8} + \frac{6}{8}$$ ✓

(b) (*6 pts.*) Find the numerical value of I_1.



this circuit include at left the circuit given



clearly: $V = -16$ volts

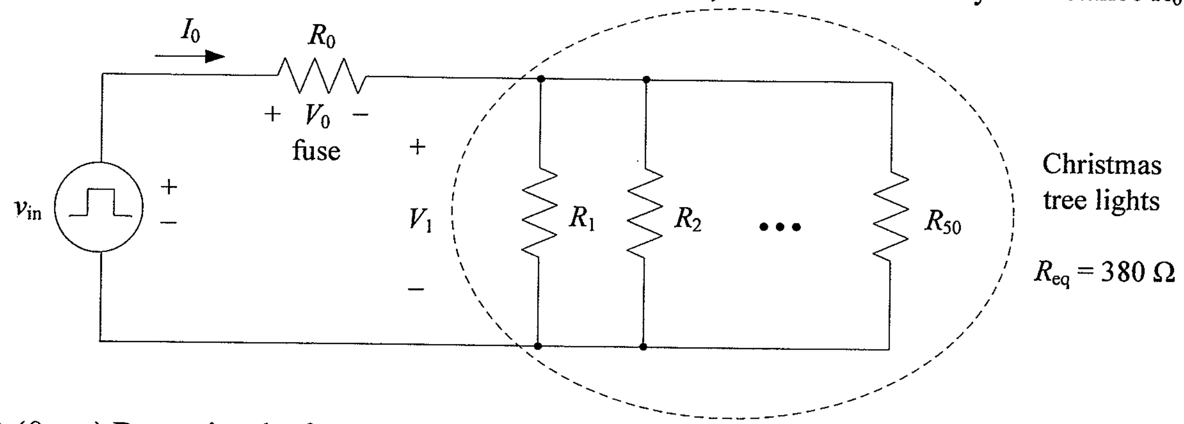
$$\Rightarrow I_1 = -\frac{1}{8}(-16) + \frac{6}{8} = \frac{22}{8}$$

$I_1 = \boxed{22/8\ A = 2.75A}$

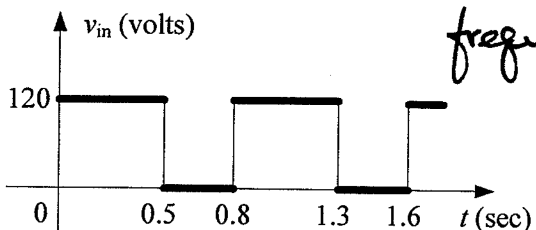
Loui

Problem 6 (22 *points*)

A voltage source produces the pulse train $v_{in}(t)$ whose period is 0.8 second (sec) shown below. The voltage source is connected to a strand of 50 Christmas tree lights. The light bulbs are modeled by identical parallel resistances $R_1 = R_2 = \ldots = R_{50} = r$. The equivalent resistance R_{eq} of the light bulbs is 380 Ω. For safety, there is a thermal fuse, which is modeled by a resistance R_0.



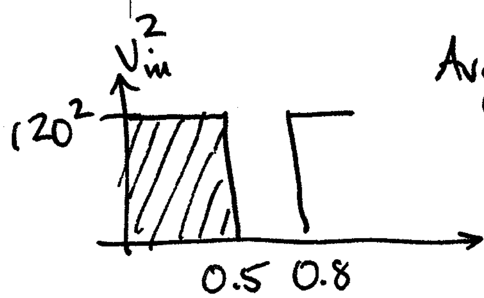
(a) (*8 pts.*) Determine the frequency of v_{in} and its root-mean-square value v_{rms}. **Show your work.**



$$\text{frequency} = \frac{1}{\text{period}} = \frac{1}{0.8} = 1.25$$

frequency of v_{in} = $\boxed{1.25\ \text{Hz}}$

v_{rms} = $\boxed{94.9\text{V}}$



$$\text{Avg}(V_{in}^2) = \frac{120^2 \times 0.5}{0.8} = 9000$$

$$V_{rms} = \sqrt{9000} = 94.9\text{V}$$

(b) (*5 pts.*) Determine the value of each light bulb resistance r. **Show your work.**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{50}} = \frac{50}{r}$$ since the light bulbs are identical.

Thus $r = 50 R_{eq} = 50 \times 380 = 19000\ \Omega$

r = $\boxed{19\ k\Omega}$

(c) (*9 pts.*) If a light bulb is replaced by a short (ideal wire), the fuse melts and breaks the circuit. When v_{in} = 120 V, determine the value of R_0 so that the fuse melts when it absorbs 250 joules of electrical energy within 0.1 sec. (Hint: first determine V_0.) **Show your reasoning.**

The short makes $V_1 = 0$, so by KVL, $V_0 = V_{in} - V_1 = V_{in}$. The power at the fuse is $V_0 I_0 = \frac{V_0^2}{R_0} = \frac{V_{in}^2}{R_0} = \frac{120^2}{R_0} = \frac{250\text{J}}{0.1\text{ sec}}$

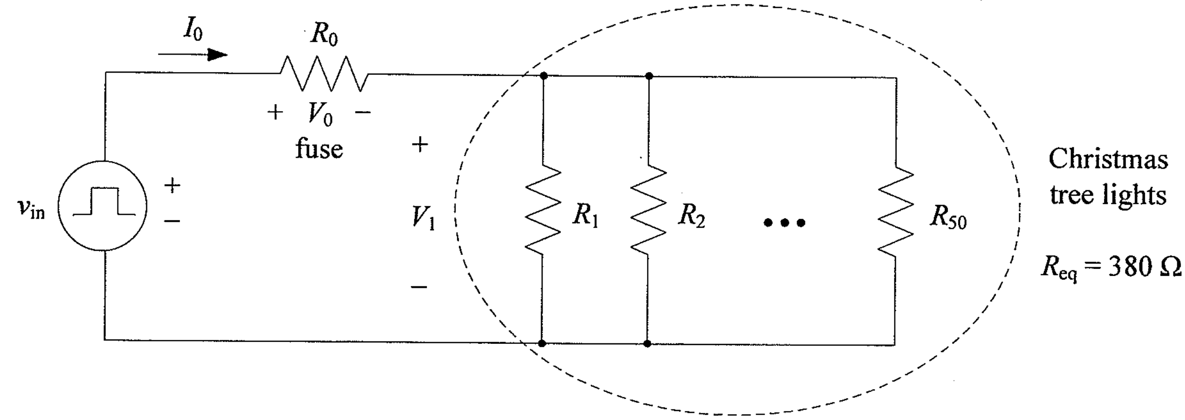
when $R_0 = \frac{120^2 \times 0.1}{250} = 5.76\ \Omega$

R_0 = $\boxed{5.76\ \Omega}$

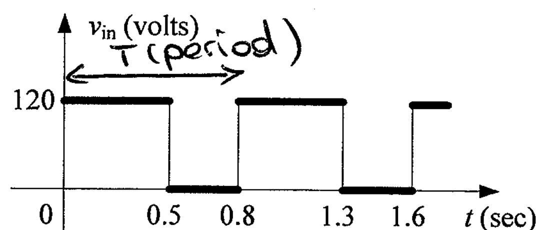
Brunet

Problem 6 (22 *points*)

A voltage source produces the pulse train $v_{in}(t)$ whose period is 0.8 second (sec) shown below. The voltage source is connected to a strand of 50 Christmas tree lights. The light bulbs are modeled by identical parallel resistances $R_1 = R_2 = \ldots = R_{50} = r$. The equivalent resistance R_{eq} of the light bulbs is 380 Ω. For safety, there is a thermal fuse, which is modeled by a resistance R_0.



(a) (*8 pts.*) Determine the frequency of v_{in} and its root-mean-square value v_{rms}. **Show your work.**

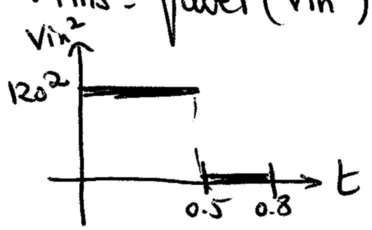


$1/T = 1/0.8$

frequency of v_{in} = $\boxed{1.25\text{ Hz}}$

v_{rms} = $\boxed{94.87\text{ volt}}$

$V_{rms} = \sqrt{\text{aver}(V_{in}^2)}$



$$\Rightarrow V_{rms} = \sqrt{\frac{120^2 \times 0.5}{0.8}} = 120\sqrt{\frac{5}{8}}$$

(b) (5 *pts.*) Determine the value of each light bulb resistance r. **Show your work.**

r all equals, and in parallel

$\Rightarrow R_{eq} = \frac{r}{50} = 380 \Rightarrow r = 380 \times 50$

r = $\boxed{19\text{ k}\Omega}$

(c) (9 *pts.*) If a light bulb is replaced by a short (ideal wire), the fuse melts and breaks the circuit. When $v_{in} = 120$ V, determine the value of R_0 so that the fuse melts when it absorbs 250 joules of electrical energy within 0.1 sec. (Hint: first determine V_0.) **Show your reasoning.**

a light bulb ≡ short $\Rightarrow V_1 = 0$; so $V_0 = V_{in} = 120$ volt.

1) the power absorbed by fuse is $P = \frac{E}{t} = \frac{250}{0.1}$ $\Rightarrow \frac{120^2}{R_0} = \frac{250}{0.1}$

2) it is also: $P = \frac{V_{in}^2}{R_0} = \frac{120^2}{R_0}$ $\Rightarrow R_0 = 120^2 \times 0.1 / 250$

R_0 = $\boxed{5.76\ \Omega}$