ECE 110

Professors Brunet and Loui

November 17, 2008

HOUR EXAMINATION #3

1) Write your:

Last Name (use capital letters): Solutions (2 versions,

First Name (use capital letters):

Signature: one from each professor)

2) Write your name and section at the back of the test.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD

Make sure to write your name AGAIN at the top of every page of your exam.

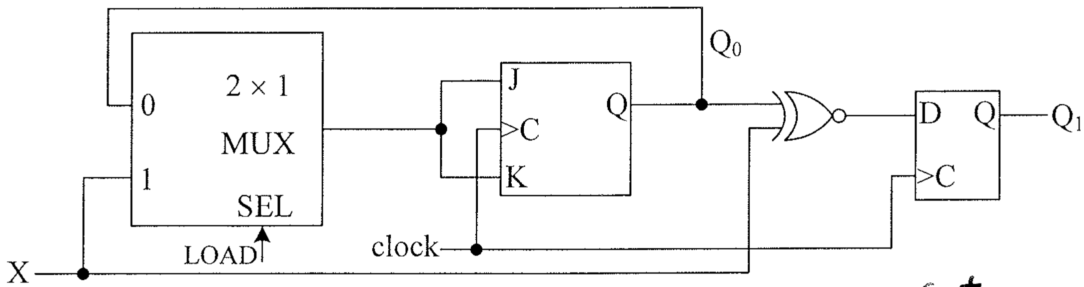
A. Write or print clearly. Answer each problem on the exam itself. If you need extra paper, there is an extra sheet at the end of this exam. Clearly identify the problem number on any additional pages. The decimal/ binary/ hexadecimal table, the Flip-flop characteristic tables, the ASCII Code, the Morse Code alphabet, the USPS Code, and numbers and properties for log base 2 are also attached to the exam.

B. In order to receive partial or full credit, **you must show all your work**, e.g., your solution process, the equation(s) that you use, the values of the variables used in the equation(s), etc. You must also include the unit of measurement in each answer.

Students caught cheating on this exam will earn a grade of F for the entire course. Other penalties may include suspension and/or dismissal from the university.

M. Loui

Problem 1 *(20 points)*



$Q_1^+ = D = \overline{X \oplus Q_0}$

a) *[14 pts.]* Complete the table for the given circuit.

When LOAD=0, $J = K = Q_0$

When LOAD=1, $J=K=X$

LOAD	Q_0	X	J	K	Q_0^+	Q_1^+
0	0	0	0	0	0	1
0	0	1	0	0	0	0
0	1	0	1	1	0	0
0	1	1	1	1	0	1
1	0	0	0	0	0	1
1	0	1	1	1	1	0
1	1	0	0	0	1	0
1	1	1	1	1	0	1

When JK=00, $Q_0^+ = Q_0$

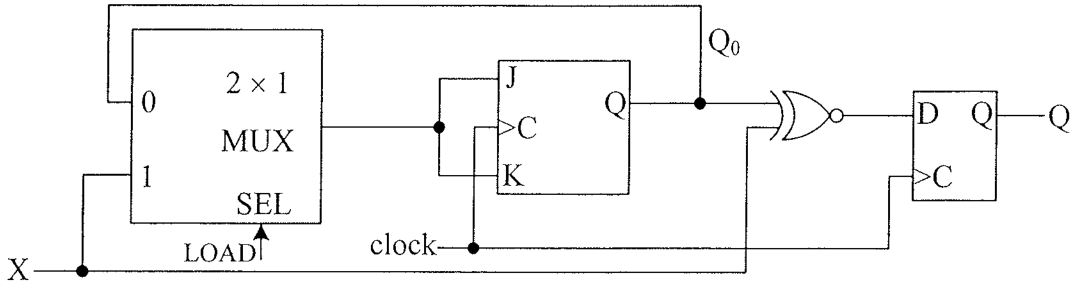
When JK=11, $Q_0^+ = \overline{Q_0}$

b) *[6 pts.]* Using your results in **a)** and assuming LOAD = 1, fill out the table below for consecutive clock pulses. Initially $Q_0(0) = 0$ and $Q_1(0) = 0$.

clock pulse	0	1	2	3
Q_0	0	1	0	0
Q_1	0	0	1	1
X	1	1	0	0

M.-C. Brunet

Problem 1 *(20 points)*



a) *[14 pts.]* Complete the table for the given circuit.

LOAD = 0, J = K = Q_0

LOAD = 1, J = K = X

LOAD	Q_0	X	J	K	Q_0^+	Q_1^+
0	0	0	0	0	0	1
0	0	1	0	0	0	0
0	1	0	1	1	0	0
0	1	1	1	1	0	1
1	0	0	0	0	0	1
1	0	1	1	1	1	0
1	1	0	0	0	1	0
1	1	1	1	1	0	1

$Q_1^+ = \overline{Q_0 \oplus X}$

looking here

Q_0^+

b) *[6 pts.]* Using your results in **a)** and assuming LOAD = 1, fill out the table below for consecutive clock pulses. Initially $Q_0(0) = 0$ and $Q_1(0) = 0$.

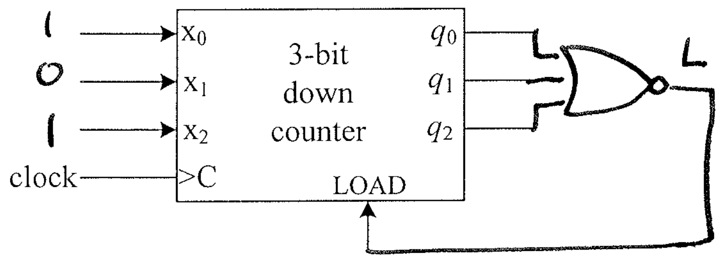
clock pulse	0	1	2	3
Q_0	0	1	0	0
Q_1	0	0	1	1
X	1	1	0	0

$Q_1^+ = \overline{Q_0 \oplus X}$

M. Loui

Problem 2 *(15 points)* You will design and analyze a mod-6 counter whose state sequence is $(q_2\, q_1\, q_0)$ = 101, 100, 011, 010, 001, 000, 101, 100, …

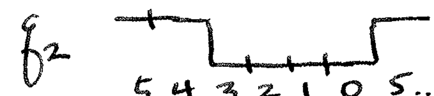
a) *[8 pts.]* Using a 3-bit down-counter with LOAD function, introduce one or more gates and constant inputs (0 or 1) to produce the specified counting sequence.



For one state $(q_2 q_1 q_0) = 000$, $L = 1$, and the next state is loaded from $(x_2\, x_1\, x_0) = 101$

b) *[7 pts.]* Suppose the clock period is 3 ms. What are the periods of q_0, q_1, and q_2?

Period of q_0 = $\boxed{6\text{ ms}}$ Period of q_1 = $\boxed{18\text{ ms}}$ Period of q_2 = $\boxed{18\text{ ms}}$



q_2 repeats after 6 clock cycles. So does q_1.

Problem 3 *(15 points)* Define $g(t) = \cos(6\pi t)$ and $h(t) = \cos(14\pi t)$, where t is in seconds. The frequency of $g(t)$ is 3 Hz, and the frequency of $h(t)$ is 7 Hz.

a) *[4 pts.]* Find a sampling frequency f_{s1} such that $f_{s1} > 7$ Hz for which $h(t)$ aliases to $g(t)$.

Frequency of $g(t)$ is $f_g = 3$ Hz, of $h(t)$ is $f_h = 7$ Hz

$f_g = |f_{s1} - f_h|$ when $f_{s1} = 10$ Hz

b) *[5 pts.]* Find a cosine whose frequency is higher than 7 Hz that also aliases to $g(t)$ when sampled at the frequency f_{s1}.

Since $3 = |10 - 13|$, frequency 13 Hz also aliases to $g(t)$: $\cos(26\pi t)$

c) *[6 pts.]* Find **two** sampling frequencies f_{s2} such that 3 Hz < f_{s2} < 7 Hz for which $h(t)$ aliases to $g(t)$. Justify your answers briefly.

$f_{s2} = 4$ Hz since $3 = |4 - 7|$, upon applying the Alias Theorem

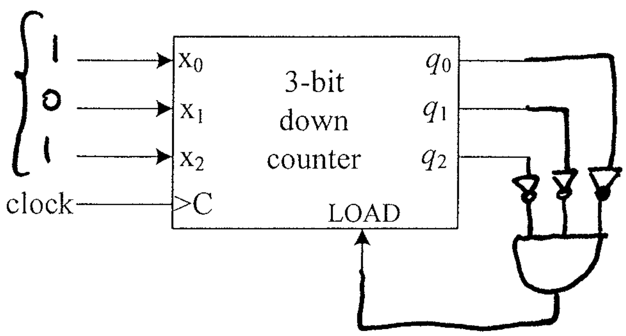
$f_{s2} = 5$ Hz also results in aliasing: By the Alias Theorem, $g(t)$ aliases to $|5-3| = 2$ Hz, and $h(t)$ aliases to $|7-5| = 2$ Hz. Thus g and h alias to each other.

M.-C. Brunet

Problem 2 *(15 points)* You will design and analyze a mod-6 counter whose state sequence is $(q_2\, q_1\, q_0) = 101, 100, 011, 010, 001, 000, 101, 100, \ldots$

a) *[8 pts.]* Using a 3-bit down-counter with LOAD function, introduce one or more gates and constant inputs (0 or 1) to produce the specified counting sequence.

when load=1 load 5



when count is 000

when count = 0 (000) do not go to 7 but to 5 -

q_2	q_1	q_0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

repeats

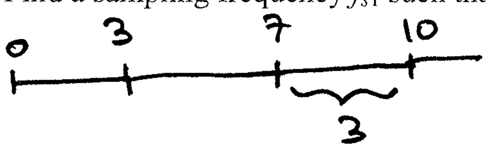
b) *[7 pts.]* Suppose the clock period is 3 ms. What are the periods of q_0, q_1, and q_2?

$2 \times 3\text{ms}$ $\qquad$ $6 \times 3\text{ms}$ $\qquad$ $6 \times 3\text{ms}$

Period of q_0 = $\boxed{6\text{ms}}$ Period of q_1 = $\boxed{18\text{ms}}$ Period of q_2 = $\boxed{18\text{ms}}$

Problem 3 *(15 points)* Define $g(t) = \cos(6\pi t)$ and $h(t) = \cos(14\pi t)$, where t is in seconds. The frequency of $g(t)$ is 3 Hz, and the frequency of $h(t)$ is 7 Hz.

a) *[4 pts.]* Find a sampling frequency f_{s1} such that $f_{s1} > 7$ Hz for which $h(t)$ aliases to $g(t)$.

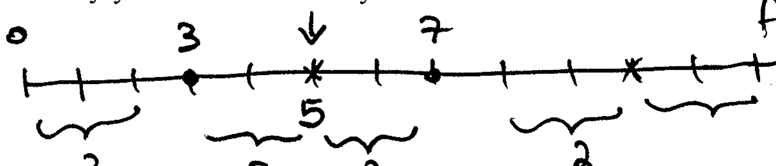


$f_{s1} = 10\text{Hz} \quad (10 - 7 = 3)$

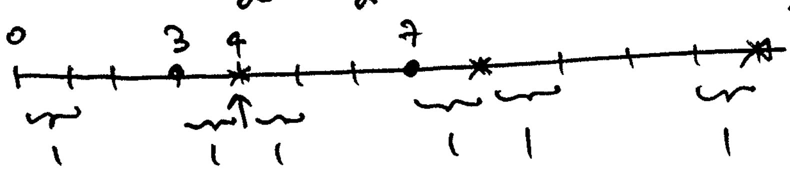
b) *[5 pts.]* Find a cosine whose frequency is higher than 7 Hz that also aliases to $g(t)$ when sampled at the frequency f_{s1}.

$f_s = 10 + 3 = 13\text{Hz} \Rightarrow \cos(26\pi t)$

c) *[6 pts.]* Find **two** sampling frequencies f_{s2} such that 3 Hz < f_{s2} < 7 Hz for which $h(t)$ aliases to $g(t)$. Justify your answers briefly.



$f_{s2} = 5\text{Hz}$; alias sequence: $(2, 3, 7, 8, 12, \ldots)$



$f_{s2} = 4\text{Hz}$; alias sequence: $(1, 3, 5, 7, 9, 11, \ldots)$

M. Loui

Problem 4 *(20 points)* A **prefix-free** code was designed for four symbols A, B, C and R. The codeword for C has been lost, but you know that the average length of the code is between 1.5 and 2.5 bits.

Symbol	A	B	C	R
Codeword	10	0	?	1101
Relative frequency	3/16	1/4	1/2	1/16

a) *[5 pts.]* Give the encoding for the name BARB

B A R B
0 10 1101 0

b) *[8pts.]* Let x be the length of the codeword for C. Using the average length of the code, determine the possible values for x. Show your work.

$$\text{Average length} = \underbrace{\left(\tfrac{3}{16}\cdot 2\right)}_{A} + \underbrace{\left(\tfrac{1}{4}\cdot 1\right)}_{B} + \underbrace{\left(\tfrac{1}{2}\cdot x\right)}_{C} + \underbrace{\left(\tfrac{1}{16}\cdot 4\right)}_{D}$$

$$= \frac{6}{16} + \frac{4}{16} + \frac{8x}{16} + \frac{4}{16} = \frac{14+8x}{16}$$

Since $1.5 \leq \frac{14+8x}{16} \leq 2.5$

$24 \leq 14+8x \leq 40$

$10 \leq 8x \leq 26$

Since x is an integer, $x = 2$ or 3

c) *[7 pts.]* Determine the codeword for C. **Explain your reasoning in words**.

The codeword for C cannot begin with 0, which conflicts with B, or with 10, which conflicts with A, because the code is prefix-free. So the codeword for C must begin with 11. But it cannot begin with 11 or 110 because those are prefixes of the codeword for R. So the codeword for C must begin with 111, and since we know from part (b) that its length is 2 or 3 bits, the codeword must be 111.

111

M.-C. Brunet

Problem 4 *(20 points)* A **prefix-free** code was designed for four symbols A, B, C and R. The codeword for C has been lost, but you know that the average length of the code is between 1.5 and 2.5 bits.

Symbol	A	B	C	R
Codeword	10	0	?	1101
Relative frequency	3/16	1/4 $= 4/16$	1/2 $= 8/16$	1/16

a) *[5 pts.]* Give the encoding for the name BARB

$$\boxed{\underbrace{0}_{B}\underbrace{10}_{A}\underbrace{1101}_{R}\underbrace{0}_{B}}$$

b) *[8pts.]* Let x be the length of the codeword for C. Using the average length of the code, determine the possible values for x. Show your work.

Lower $\Rightarrow$ $\dfrac{3\times 2 + 4\times 1 + 8x + 1\times 4}{16} = \dfrac{14+8x}{16}$

$1.5 \leq \dfrac{14+8x}{16} \leq 2.5 \Rightarrow 24 \leq 14+8x \leq 40$

$\Rightarrow 10 \leq 8x \leq 26 \Rightarrow 1.25 \leq x \leq 3.25$

$\Rightarrow$ $\boxed{x \text{ can be } 2 \text{ or } 3 \text{ bits}}$

c) *[7 pts.]* Determine the codeword for C. **Explain your reasoning in words.**

If $x = 2$; cannot start with 0 (coding for B) since prefix free!
; if starts with 1 : cannot be 10 (coding for A)
: cannot be 11 (prefix of coding for R)

$\Rightarrow$ x must be 3

must start with 11 (reasons above)
110 : cannot since it is prefix of 1101 (coding for R)
111 : only possible coding left $\boxed{111}$

M. Louie

Problem 5 *(15 points)* A facsimile (fax) machine digitizes an image using pixels that are 1/8 mm wide and 1/4 mm high. Each pixel color is quantized with one bit, white (1) or black (0).

a) *[6 pts.]* Determine the time, in seconds, required to fax a document that is 8.5 inches wide and 11 inches high, without data compression, over a telephone line that allows digital data to be transmitted at 30,000 bits per second. Use the conversion factor 1 inch = 25.4 mm. (**Hint**: The width is 1728 pixels.)

$$\text{Height} = \frac{11 \text{ in.} \times 25.44 \text{ mm/in.}}{\frac{1}{4} \text{ mm/pixel}} = 1118 \text{ pixels (rounded up)}$$

Note one bit per pixel.

$$\text{Time} = \frac{1728 \times 1118 \text{ bits}}{30{,}000 \text{ bits/sec}} = 64.4 \text{ sec}$$

Time = $\boxed{64.4 \text{ sec}}$

b) *[5 pts.]* Suppose a run-length code is used in which the length of a run from one to seven is represented in binary with 3 bits, preceded by the color bit. For instance, a run of five black pixels is encoded 0 1 0 1. A run of twelve black pixels is encoded as consecutive runs of lengths seven and five: 0 1 1 1 0 1 0 1. Determine the compression ratio for an 8.5 inch row of pixels that all have the same color.

A row of 1728 pixels has 246 runs of 7 pixels, plus one more run of 6.

$$R_{comp} = \frac{1728}{247 \text{ runs} \times 4 \text{ bits/run}} = 1.75$$

Compression ratio = $\boxed{1.75}$

c) *[4 pts.]* Propose a modification to the run-length code to increase the compression ratio for long runs of the same color. (**Hint**: What could code words 0 0 0 0 and 1 0 0 0 mean?)

There are many solutions. For example, define 0000 to denote a run of 24 black pixels and 1000 to denote a run of 24 white pixels. For a long run of the same color, the compression ratio is $\frac{24 \text{ bits}}{4 \text{ bits}} = 6$.

M.-C. Brunet.

Problem 5 *(15 points)* A facsimile (fax) machine digitizes an image using pixels that are 1/8 mm wide and 1/4 mm high. Each pixel color is quantized with one bit, white (1) or black (0).

a) *[6 pts.]* Determine the time, in seconds, required to fax a document that is 8.5 inches wide and 11 inches high, without data compression, over a telephone line that allows digital data to be transmitted at 30,000 bits per second. Use the conversion factor 1 inch = 25.4 mm. (**Hint**: The width is 1728 pixels.)

8.5 inches × 11 inches = 8.5 × 25.4 × 8 pixels × 11 × 25.4 × 4 pixels

= 1728 × 1118 bits (1 bit, 1 pixel)

$\Rightarrow \dfrac{1728 \times 1118}{30{,}000}$ seconds

Time = $\boxed{\text{64.4 seconds}}$

b) *[5 pts.]* Suppose a run-length code is used in which the length of a run from one to seven is represented in binary with 3 bits, preceded by the color bit. For instance, a run of five black pixels is encoded 0 1 0 1. A run of twelve black pixels is encoded as consecutive runs of lengths seven and five: 0 1 1 1 0 1 0 1. Determine the compression ratio for an 8.5 inch row of pixels that all have the same color. original data is 1728 bits for 1 row

1728 pixels = 246×7 + 6

coding (example) $\underbrace{0111\ 0111\ \ldots\ 0111}_{246\ \text{times}}$ 0101 ⇒ 247×4 bits

compression = $\dfrac{1728}{247\times4} = \dfrac{1728}{988}$

Compression ratio = $\boxed{1.75}$

c) *[4 pts.]* Propose a modification to the run-length code to increase the compression ratio for long runs of the same color. (**Hint**: What could code words 0 0 0 0 and 1 0 0 0 mean?)

If we use 000 to code number 8, (code for 0 is never needed or used) then it will compress even more.

example: 1728 pixels = 216×8

coding $\underbrace{0000/0000/0000\ \ldots\ 0000}_{216\ \text{times}}$ ⇒ 216×4 bits

compression = $\dfrac{1728}{216\times4} = \dfrac{1728}{864} = 2$ (instead of 1.75)

M. loui

Problem 6 *(15 points)*

For easy error detection and correction, it was decided to send a message of two ASCII symbols three times. The message below was received. There is one error.

1 0 0 0 1 1 0 0 1 1 0 0 1 0 1 0 0 0 1 1 0 0 1 1 1 0 1 0 1 0 0 0 1 1 0 0 1 1 0 0 1 0

groups of 7 bits

a) *[5 pts.]* Indicate which bit is wrong (circle it). Show your work.

0110010 0111010 0110010

These groups of 7 bits are identical except for the single circled bit.

b) *[6 pts.]* Give the 14-bit message that was intended to be sent (**sequence of bits**).

1000110 0110010

c) *[4 pts.]* Give the two **symbols** that were intended to be sent.

F2

M.-C. Brunet

Problem 6 *(15 points)*

For easy error detection and correction, it was decided to send a message of two ASCII symbols three times. The message below was received. There is one error.

1 0 0 0 1 1 0 0 1 1 0 0 1 0 1 0 0 0 1 1 0 0 1 1 1 0 1 0 1 0 0 0 1 1 0 0 1 1 0 0 1 0

a) *[5 pts.]* Indicate which bit is wrong (circle it). Show your work.

1000110 0110010

1000110 011$\boxed{1}$010 ← wrong bit since there is only one bit error.

1000110 0110010

b) *[6 pts.]* Give the 14-bit message that was intended to be sent (**sequence of bits**).

1000110 0110010

c) *[4 pts.]* Give the two **symbols** that were intended to be sent.

(from the ASCII Table).

F 2