ECE 110
Professors Brunet and Loui **November 16, 2009**

HOUR EXAMINATION #3

1) Write your official (*not a nickname*):
Last Name (use capital letters): Solutions (2 versions,
First Name (use capital letters): from each professor)
Signature:____________________________________

2) Write your name and <u>section</u> at the back of the test.

<u>DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD</u>

<u>Make sure to write your name AGAIN at the top of every page of your exam.</u>

A. Write or print clearly. Answer each problem on the exam itself. If you need extra paper, there is an extra sheet at the end of this exam. Clearly identify the problem number on any additional pages. The decimal/ binary/ hexadecimal table, the Flip-flop characteristic tables, the ASCII Code, the Morse Code alphabet, the USPS Code, and numbers and properties for log base 2 are also attached to the exam.

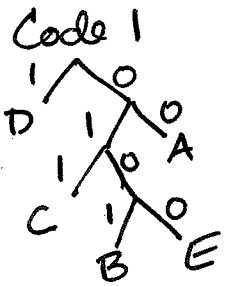
B. In order to receive partial or full credit, **you must show all your work,** e.g., your solution process, the equation(s) that you use, the values of the variables used in the equation(s), etc. You must also include the unit of measurement in each answer.

Students caught cheating on this exam will earn a grade of F for the entire course. Other penalties may include suspension and/or dismissal from the university.

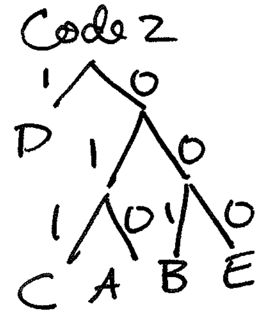
Loui

Problem 1 *(25 points)*

Consider five symbols with given relative frequencies. Two different codes were designed to achieve compression over a fixed length code.



Symbol	A	B	C	D	E
Frequency	3/15	2/15	3/15	6/15	1/15
Code 1	00	0101	011	1	0100
Code 2	010	001	011	1	000



(a) *(5 pts)* Show that the average code word length for Code 1 is 2.2 bits/symbol.

Average length $= \frac{1}{15}(\overset{A}{2\times 3} + \overset{B}{2\times 4} + \overset{C}{3\times 3} + \overset{D}{6\times 1} + \overset{E}{1\times 4})$

$= 2.2$ bits/symbol

(b) *(5 pts)* Terry, Kris and Sam computed the Entropy. Without computing it yourself, check which result(s) might be correct. (check all that apply). Justify your answer.

☑ $E_{Terry} = 2.05$ ☐ $E_{Kris} = 2.32$ ☑ $E_{Sam} = 1.98$

Justify:

The entropy is less than the length of any code.
$E_{Terry} < 2.2$ and $E_{Sam} < 2.2$, but $E_{Kris} > 2.2$

(c) *(5 pts)* What is the compression ratio for Code 1 as compared with a fixed length code?

☐ 0.70 ☐ 0.73 ☑ 1.36 ☐ 1.42 ☐ other

A fixed length code for five symbols must use (at least) 3 bits since $2^2 = 4 < 5 < 2^3 = 8$. By part (a), $\frac{3}{2.2} = 1.36$

(d) *(5 pts)* Which of Code 1 or Code 2 is prefix-free?

☐ Neither ☐ Only Code 1 ☐ Only Code 2 ☑ Both

See code trees above.

(e) *(5 pts)* For data compression purposes, which of Code 1 or Code 2 is preferable, and why?

Average length of Code 2 $= \frac{1}{15}(\overset{D}{1\times 6} + \overset{\text{others}}{3\times 9}) = \frac{6+27}{15}$

$= 2.2$ bits/symbol

They are equally preferable because they achieve the same compression ratio.

Bruenet

Problem 1 *(25 points)*
Consider five symbols with given relative frequencies. Two different codes were designed to achieve compression over a fixed length code.

Symbol	A	B	C	D	E
Frequency	3/15	2/15	3/15	6/15	1/15
Code 1	00	0101	011	1	0100
Code 2	010	001	011	1	000

(a) *(5 pts)* Show that the average code word length for Code 1 is 2.2 bits/symbol.

$$L_{aver} = \frac{3}{15}\times 2 + \frac{2}{15}\times 4 + \frac{3}{15}\times 3 + \frac{6}{15}\times 1 + \frac{1}{15}\times 4$$

$$= \frac{6+8+9+6+4}{15} = \frac{33}{15} = 2.2$$

(b) *(5 pts)* Terry, Kris and Sam computed the Entropy. Without computing it yourself, check which result(s) might be correct. (check all that apply). Justify your answer.

☑ $E_{Terry} = 2.05$ ☐ $E_{Kris} = 2.32$ ☑ $E_{Sam} = 1.98$

Justify: The entropy is the minimum; so in particular it must be less than 2.2 (from (a)) -

(c) *(5 pts)* What is the compression ratio for Code 1 as compared with a fixed length code?

☐ 0.70 ☐ 0.73 ☑ 1.36 ☐ 1.42 ☐ other

Fixed length code : 5 symbols (A → E) ⇒ 3 bits for each

$$\text{compression} = \frac{3}{2.2} = 1.36$$

(d) *(5 pts)* Which of Code 1 or Code 2 is prefix-free?

☐ Neither ☐ Only Code 1 ☐ Only Code 2 ☑ Both

A code tree can be generated for both -

(e) *(5 pts)* For data compression purposes, which of Code 1 or Code 2 is preferable, and why?

$$L_{aver}(\text{code } 2) = \frac{3}{15}\times 3 + \frac{2}{15}\times 3 + \frac{3}{15}\times 3 + \frac{6}{15}\times 1 + \frac{1}{15}\times 3 = \frac{33}{15} = 2.2$$

Code 2 has the same average code word length as Code 1 ⇒ same compression -

⇒ None is preferable -

Loui

Problem 2 *(10 points)*
A DVD has a capacity of 4.7 GB. A DVD can be used for CD-quality music, sampled at 44.1 kHz, with 16 bits of data per sample (for stereo), and 8 bits per byte.

(a) *(6 pts)* Calculate the maximum number of hours of music that can be recorded in this format on one DVD, with no data compression.

$$\frac{4.7 \times 2^{30} \text{ bytes}}{(44.1 \times 10^3 \text{ samples/sec})(2 \text{ bytes/sample})(3600 \text{ sec/hr})}$$

$= 15.89$ hr

(b) *(4 pts)* Suppose a Cyclic Redundancy Check (CRC) code is used to correct single-bit errors, as follows. For every run of **fifteen** consecutive data bytes, the code introduces 24 additional bits: one parity bit for each data byte, and then **nine** more parity bits, one for each bit position. (An actual CD uses a different error-correcting code.) Using the result of part (a), calculate the maximum number of hours of music that can be recorded with the CRC code on one DVD.

For every 15 data bytes, there are 3 additional bytes for the 24 error correction bits. So each block of 15 data bytes occupies 18 bytes on the DVD.
Total capacity is $15.89 \times \frac{15}{18} = 13.24$ hr

Problem 3 *(10 points)*
Let $u(t) = \cos(6\pi t)$ and $v(t) = \cos(16\pi t)$, where t is in seconds.

(a) *(5 pts)* Find a sampling frequency f_{s1} between the frequencies if u and v such that $v(t)$ aliases to $u(t)$. Show your work.

Frequencies of u and v are $f_u = \frac{6\pi}{2\pi} = 3$ Hz, $f_v = \frac{16\pi}{2\pi} = 8$ Hz.
By the Alias Theorem, when v is sampled at frequency $f_{s1} = 5$ Hz, u is an alias since $f_u = f_v - f_{s1}$.

(b) *(5 pts)* Determine the **maximum** sampling frequency f_{s2} such that $v(t)$ aliases to $u(t)$. Explain your reasoning.

By the Alias Theorem, v alias to u when sampled at frequency $f_{s2} = 11$ Hz since $f_u = f_{s2} - f_v$. Furthermore, for any sampling frequency f_s' between 11 Hz and the Nyquist rate $2f_v = 16$ Hz, v aliases to a frequency $f_s' - f_v > f_u$. So f_{s2} is the maximum.

Bruret

2^{30} $44{,}100\text{Hz}$

Problem 2 *(10 points)*

A DVD has a capacity of 4.7 GB. A DVD can be used for CD-quality music, sampled at 44.1 kHz, with 16 bits of data per sample (for stereo), and 8 bits per byte.

2 Byte

(a) *(6 pts)* Calculate the maximum number of hours of music that can be recorded in this format on one DVD, with no data compression.

$$\frac{4.7 \times 2^{30} \text{ Bytes}}{(44{,}100 \text{ samples/sec}) \times (2 \text{ Bytes/sample}) \times (3600 \text{ sec/hour})}$$

$$= \underline{15.89 \text{ hours}}$$

(b) *(4 pts)* Suppose a Cyclic Redundancy Check (CRC) code is used to correct single-bit errors, as follows. For every run of **fifteen** consecutive data bytes, the code introduces 24 additional bits: one parity bit for each data byte, and then **nine** more parity bits, one for each bit position. (An actual CD uses a different error-correcting code.) Using the result of part (a), calculate the maximum number of hours of music that can be recorded with the CRC code on one DVD.

More bits are used for correction (less data for music itself) $\Rightarrow$ lower number of hours than (a) by a ratio:

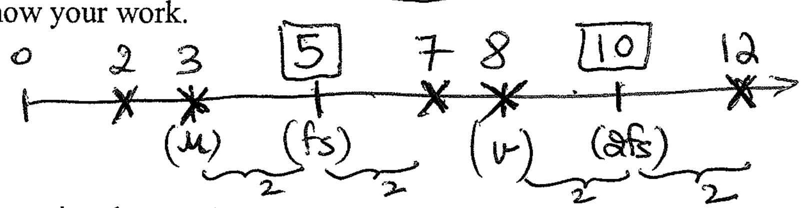
$$\frac{15B}{15B+24} = \frac{15 \times 8}{15 \times 8 + 24} = \frac{15}{18} = 83.33\%$$

$$\Rightarrow \#\text{hours} = 83.33\% \, (15.89) = \underline{13.24 \text{ hours}}$$

Problem 3 *(10 points)*

Let $u(t) = \cos(6\pi t)$ and $v(t) = \cos(16\pi t)$, where t is in seconds.

(a) *(5 pts)* Find a sampling frequency f_{s1} between the frequencies if u and v such that $v(t)$ aliases to $u(t)$. Show your work.



(X: aliases)

$\underline{f_{s1} = 5\text{ Hz}}$

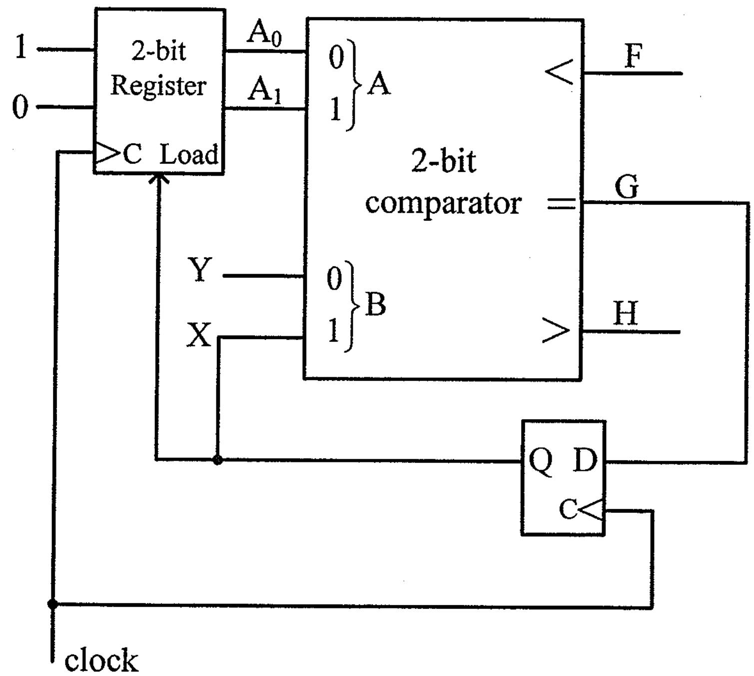
(b) *(5 pts)* Determine the **maximum** sampling frequency f_{s2} such that $v(t)$ aliases to $u(t)$. Explain your reasoning.

$u \rightarrow 3\text{ Hz}$; $NR = 6\text{ Hz}$ | if $f_{s2} > 16\text{ Hz}$; no aliasing

$v \rightarrow 8\text{ Hz}$; $NR = 16\text{ Hz}$ | $\Rightarrow f_{s2}$ must be $\leq 16\text{ Hz}$.

Louie

Problem 4 *(15 points)*



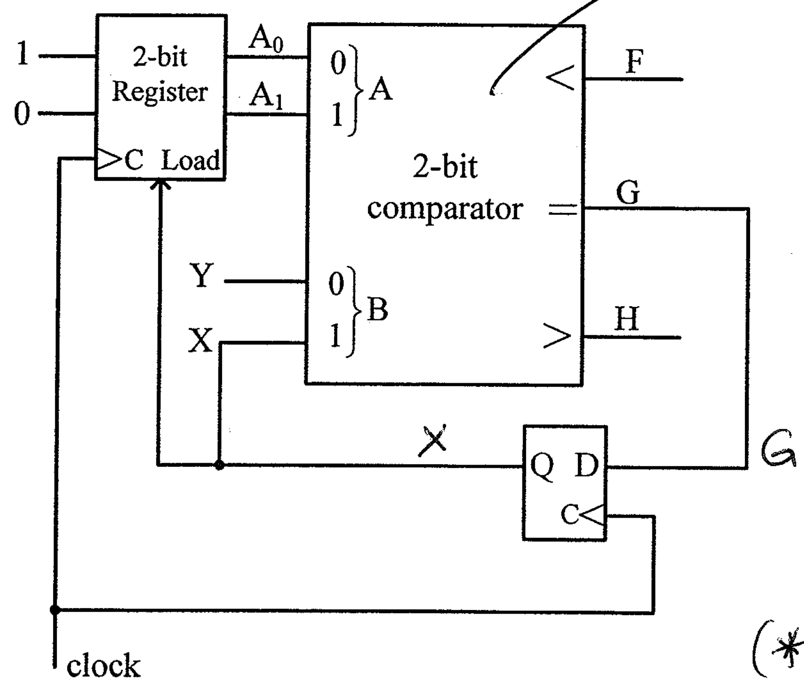
F = 1 when A < B
G = 1 when A = B
H = 1 when A > B

Fill out the table below for consecutive clock pulses.

Pulse	A_1	A_0	X	Y	F	G	H	
0	0	0	0	1	1	0	0	00 < 01
1	0	0	0	0	0	1	0	00 = 00
2	0	0 Load	1	0	1	0	0	00 < 10
3	0	1	0	0	0	0	1	01 > 00
4	0	1	0	1	0	1	0	01 = 01

Brunet

Problem 4 *(15 points)*



compares A_1A_0 / XY

(*) | X = load to "register"
$X = 0 \Rightarrow A_1^+ A_0^+ = A_1 A_0$
$X = 1 \Rightarrow A_1^+ A_0^+ = 0\,1$

Fill out the table below for consecutive clock pulses.

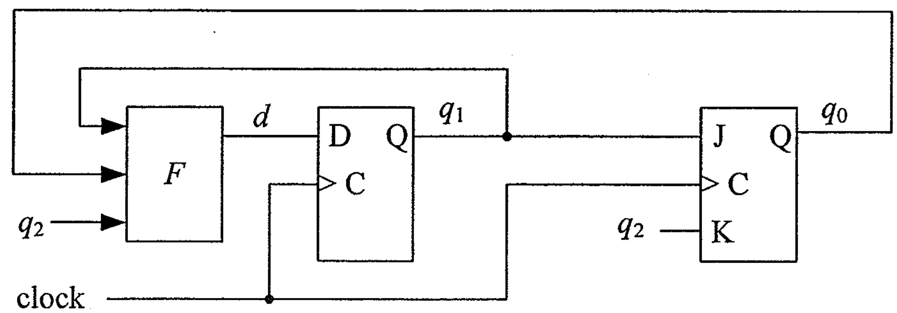
	Pulse	A_1	A_0		X	Y	F	G	H
	0	0	0	<	0	1	1	0	0
keep	1	0	0	=	0	0	0	1	0
keep	2	0	0	<	1	0	1	0	0
load!	3	0	1	>	0	0	0	0	1
keep	4	0	1	=	0	1	0	1	0

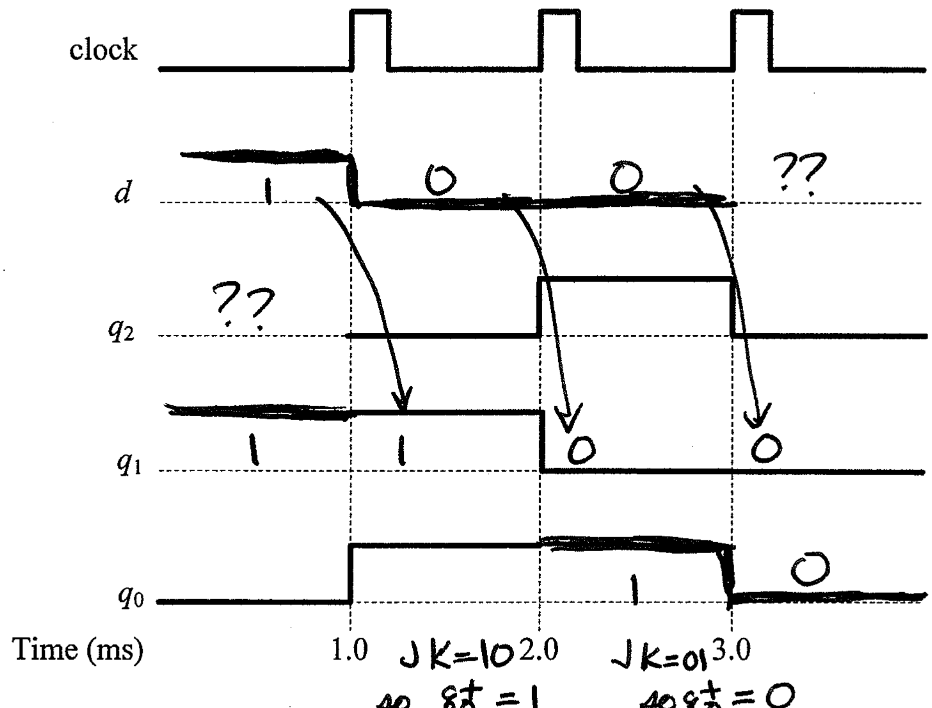
see (*)

$X^+ = G$
(D flip-flop)

Loui

Problem 5 *(20 points)*
This sequential circuit has positive-edge-triggered D and JK flip-flops. Variable q_2 is the state of a flip-flop that is not shown, but connected to the same clock signal. The combinational circuit F implements d as a Boolean function of q_2, q_1, and q_0.





(a) *(16 pts.)* Part of the timing diagram is given. Determine the entire waveform for d, and the remainder of the waveforms for q_2, q_1, and q_0. Assume that the delay in F is negligible. If there is insufficient information to determine a value, write "??" in the appropriate space.

(b) *(4 pts.)* **Explain your reasoning** for your answers for the values of q_2 and q_1 before time 1.0. Be sure to explain why alternative values are invalid.

Because $q_0 = 0$ before time 1.0, if $q_1 q_2 = JK =$

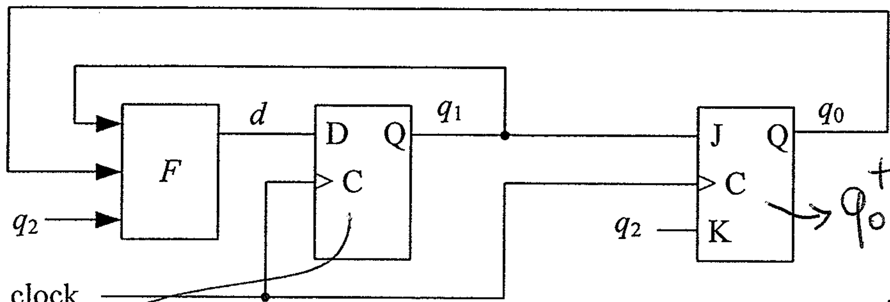
00 then $q_0^+ = q_0 = 0$ } Inconsistent with $q_0 = 1$ from 1.0 to 2.0 ms
01 $q_0^+ = 0$
10 $q_0^+ = 1$ } Consistent with $q_0 = 1$ from 1.0 to 2.0 ms
11 $q_0^+ = \bar{q}_0 = 1$

Therefore, before 1.0 ms, $q_1 = 1$ and $q_2 = 0$ or 1.

Bruenet

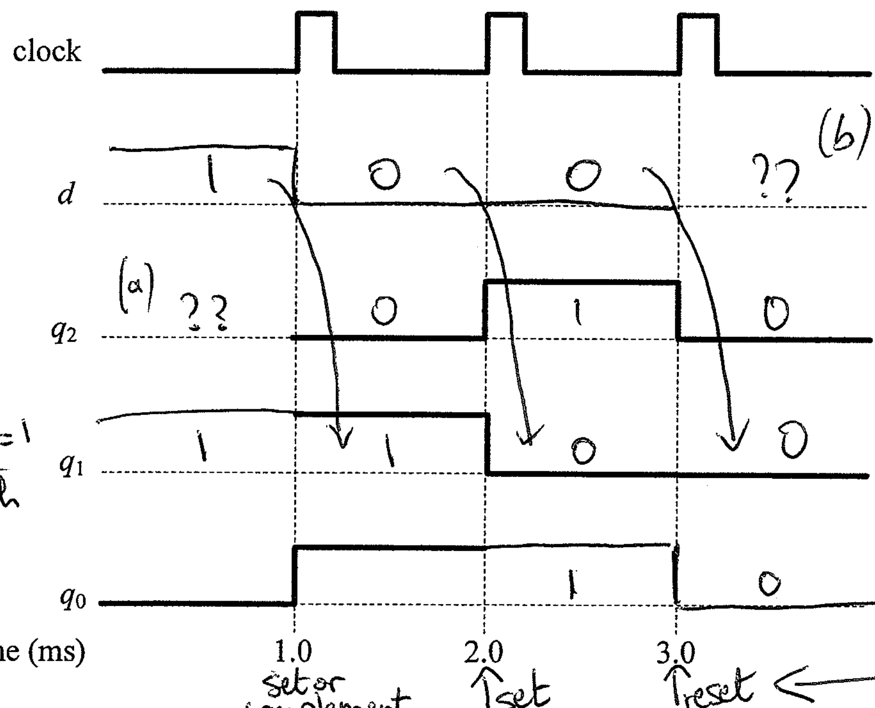
Problem 5 *(20 points)*

This sequential circuit has positive-edge-triggered D and JK flip-flops. Variable q_2 is the state of a flip-flop that is not shown, but connected to the same clock signal. The combinational circuit F implements d as a Boolean function of q_2, q_1, and q_0.



q_0^+ depends on $JK = q_1 q_2$

$q_1^+ = d$



(a) no other time with $d=1$, $q_1=1$ $\Rightarrow$ cannot know q_2 such that $F(1,0,q_2)=1$

(b) no other time with $q_2=q_1=q_0=0$ $\Rightarrow$ cannot know d such that $F(0,0,0)=d$

(a) *(16 pts.)* Part of the timing diagram is given. Determine the entire waveform for d, and the remainder of the waveforms for q_2, q_1, and q_0. Assume that the delay in F is negligible. If there is insufficient information to determine a value, write "??" in the appropriate space.

(b) *(4 pts.)* **Explain your reasoning** for your answers for the values of q_2 and q_1 before time 1.0. Be sure to explain why alternative values are invalid.

before 1.0 $q_0 = 0$ }
after 1.0 $q_0^+ = 1$ } JK must have been either 10 (set) or 11 ($\bar{q_0}$) ($q_1 q_2$)

$\Rightarrow$ in any case $J = q_1 = 1$ $K = q_2$ can be 0 or 1

Loui

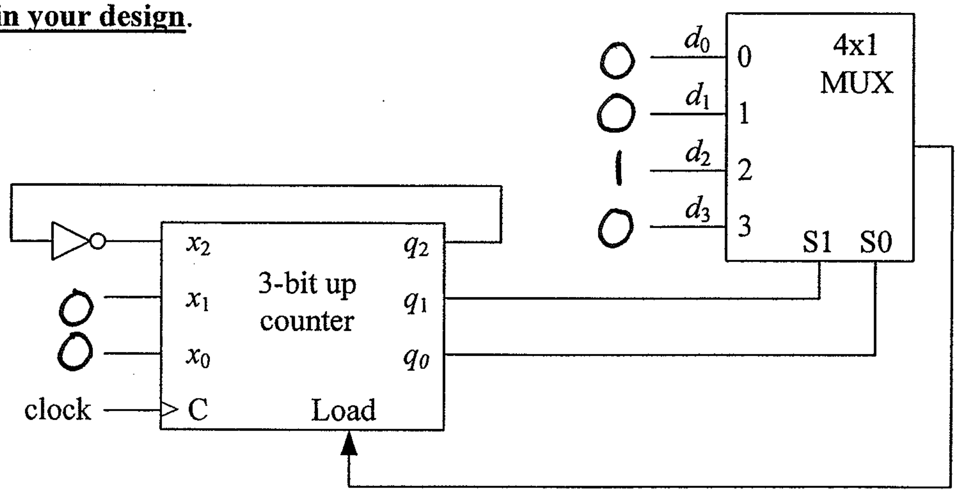
Problem 6 *(20 points)*
You will design and analyze a mod-6 counter whose state sequence is
$(q_2\, q_1\, q_0)$ = 000, 001, 010, 100, 101, 110, 000, … Note that the counting sequence is **not** the sequence of binary representations of 0, 1, 2, 3, 4, 5.

(a) *(8 pts)* If the clock period is 7 ms, what are the periods of q_0, q_1, and q_2?

Period of q_0 = $\boxed{21 \text{ ms}}$ Period of q_1 = $\boxed{21 \text{ ms}}$ Period of q_2 = $\boxed{42 \text{ ms}}$

q_0 sequence : 010010 repeats after 3 clock periods, 3×7=21
q_1 sequence : 001001 3 3×7=21
q_2 sequence : 000111 6 6×7=42

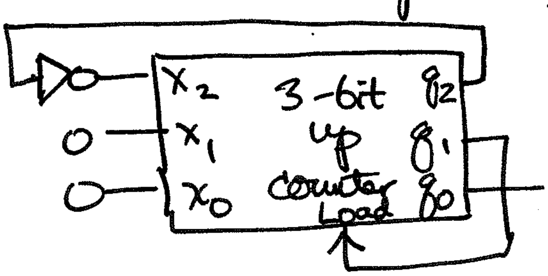
(b) *(8 pts)*] Shown below is a partial design that uses a 3-bit up counter and a 4x1 multiplexer. Specify constant inputs x_1, x_0, d_0, d_1, d_2, d_3, to produce the specified counting sequence. **Explain your design**.



To implement the state transitions 010 → 100 and 110 → 000, which occur when $(q_1 q_0) = 10$, the multiplexer selects $d_2 = 1$ as the Load signal; the counter replaces $(q_2 q_1 q_0)$ with $(\bar{q}_2 00)$.

(c) *(4 pts)* Describe a simpler design that eliminates the multiplexer. **Justify your design**.

Remove the multiplexer, connect q_1 directly to Load, and keep the rest the same. Since 011 and 111 are not part of the counting sequence, $q_1 = 1$ if and only if the current state is 010 or 110, and the next state should be $(\bar{q}_2 00)$.

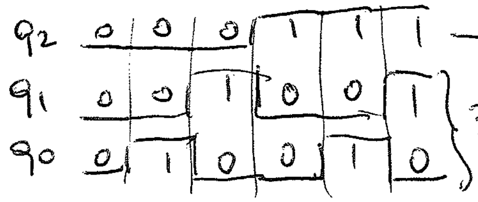


Brunet

Problem 6 *(20 points)*
You will design and analyze a mod-6 counter whose state sequence is
($q_2\, q_1\, q_0$) = 000, 001, 010, 100, 101, 110, 000, … Note that the counting sequence is **not** the sequence of binary representations of 0, 1, 2, 3, 4, 5.

(a) *(8 pts)* If the clock period is 7 ms, what are the periods of q_0, q_1, and q_2?

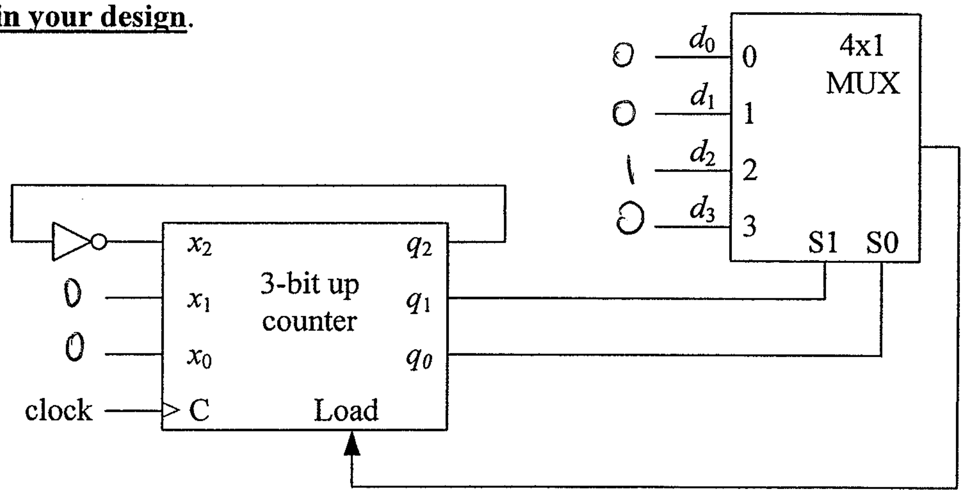
Period of q_0 = $\boxed{21ms}$ Period of q_1 = $\boxed{21ms}$ Period of q_2 = $\boxed{42ms}$



→ 6 clock periods (6x7)

3 clock periods (3x7)

(b) *(8 pts)*] Shown below is a partial design that uses a 3-bit up counter and a 4x1 multiplexer. Specify constant inputs x_1, x_0, d_0, d_1, d_2, d_3, to produce the specified counting sequence. **Explain your design**.



000 $\xrightarrow{\text{counts}}$ 001 $\xrightarrow{\text{counts}}$ 010 $\xrightarrow{(*)}$ 100 $\xrightarrow{\text{counts}}$ 101 $\xrightarrow{\text{counts}}$ 110 $\xrightarrow{(*)}$ 000

(*) when $q_1q_0 = 10$ | $q_1^+q_0^+ = 00$
$q_2^+ = \overline{q_2}$
(so $x_1 x_0 = 00$)
and Load must be 1 ⇒ $d_2 = 1$
(all others 000)

(c) *(4 pts)* Describe a simpler design that eliminates the multiplexer. **Justify your design**.

Load = 1 when $q_1q_0 = 10$ ⇒ instead of MUX, use AND gate Load = $q_1\overline{q_0}$