

ECE 110 Transparencies

Set 4 for Noon Lecture

Lippold Haken

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**These transparencies include material by
Prof. Tim Trick and Prof. Marie-Christine Brunet**

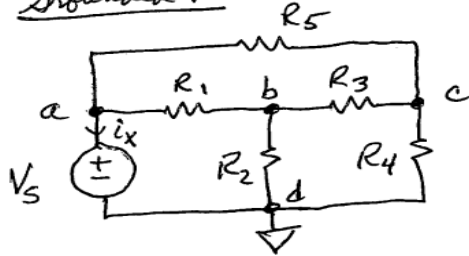
Nodal Analysis

Nodal analysis is a particular method of solving circuits. You write equations in terms of node voltages, and then solve those equations.

- 1) Label every node in the circuit, and label currents through each voltage source and current source.**
- 2) Write down any “known” voltage:**
 - Ground is 0 volts**
 - Use node voltage constraints across voltage sources**
- 3) Write KCL equations in terms of the node voltages.
For n nodes, you have $n-1$ equations.**
- 4) Solve the equations to get the unknown node voltages.**
- 5) Solve for currents through each element (if you are asked to do so) using the node voltages.**
- 6) Solve for power (if you are asked to do so).
Elements supplying energy: Negative power
Elements absorbing energy: Positive power**

Examples: How to write Nodal Analysis equations directly from the circuit diagram.

1) grounded V_S



(a) $i_x + \frac{v_a - v_b}{R_1} + \frac{v_a - v_c}{R_5} = 0$

(b)

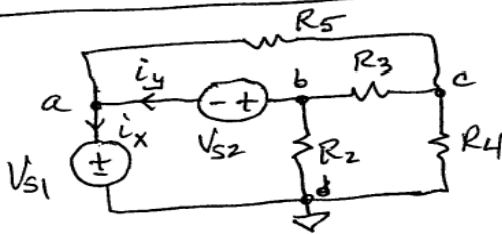
$$\frac{v_b - v_a}{R_1} + \frac{v_b}{R_2} + \frac{v_b - v_c}{R_3} = 0$$

$$\frac{v_c - v_b}{R_3} + \frac{v_c - v_a}{R_5} + \frac{v_c}{R_4} = 0$$

(c)

add constraint
 $v_a = V_S$
 and solve 2 eqs.
 in 2 unknowns.
 Return to (a) to find i_x .

2)



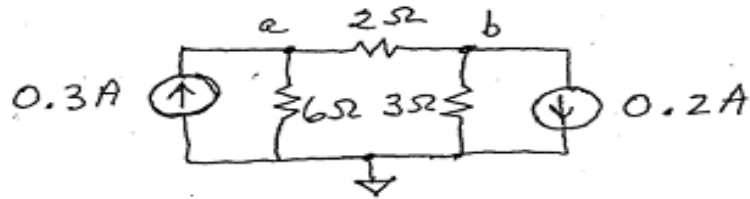
(a) $i_x - i_y + \frac{v_a - v_c}{R_5} = 0$

(b) $+i_y + \frac{v_b}{R_2} + \frac{v_b - v_c}{R_3} = 0$

(c) $\frac{v_c}{R_4} + \frac{v_c - v_b}{R_3} + \frac{v_c - v_a}{R_5} = 0$

Exercise

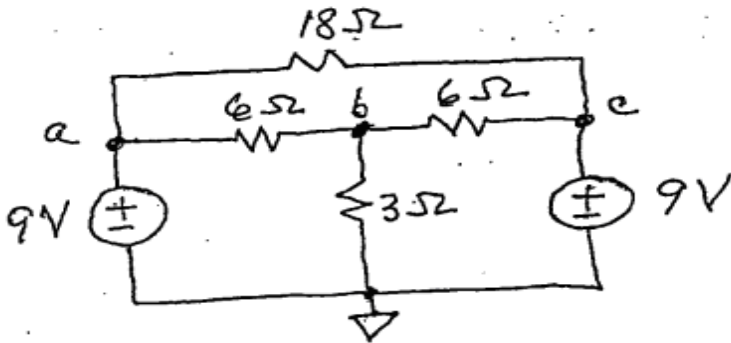
Solve for the node voltages. Verify your answers.



$$v_a = \boxed{} \quad v_b = \boxed{}$$

Quiz LH15

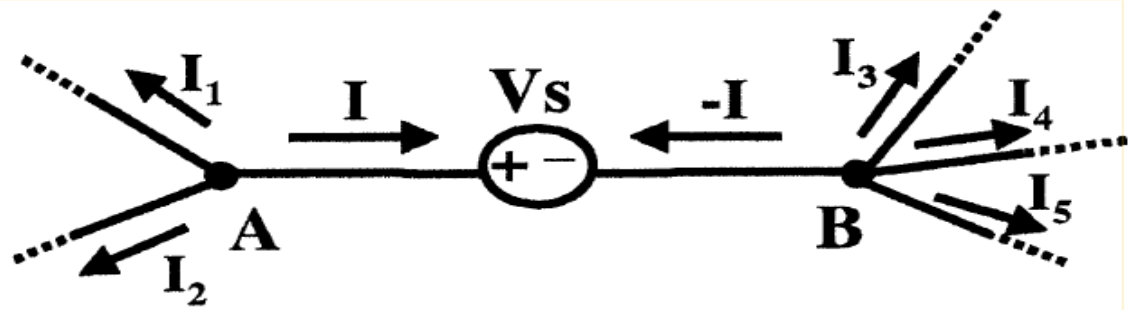
Solve for the node voltages. Verify your answers.



$$v_a = \boxed{} \quad v_b = \boxed{}$$

$$v_c = \boxed{}$$

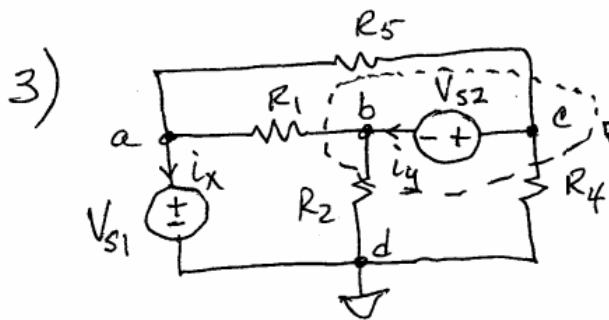
Floating Voltage Source



Neither V_A nor V_B are known!
→ 2 equations needed

sum { **KCL @ A:**
KCL @ B:
KCL @ 'AB':

Examples



$$\textcircled{a} \quad i_x + \frac{v_a - v_b}{R_1} + \frac{v_a - v_c}{R_5} = 0$$

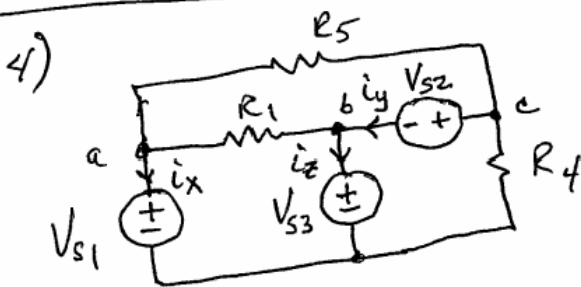
$$\textcircled{b} \quad -i_y + \frac{v_b - v_a}{R_1} + \frac{v_b}{R_2} = 0$$

$$\textcircled{c} \quad +i_y + \frac{v_c}{R_4} + \frac{v_c - v_a}{R_5} = 0$$

$\textcircled{b} + \textcircled{c}$ eliminates i_y

Since $v_a = V_{s1}$
and $v_c - v_b = V_{s2}$,

yields one eq. in one unknown.



Note: $v_a = V_{s1}$

$v_b = V_{s3}$

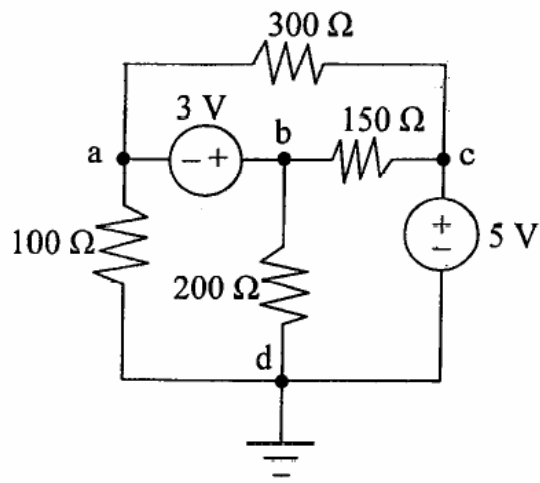
$v_c - v_b = V_{s2}$ or $v_c = V_{s2} + V_{s3}$

$$\textcircled{a} \quad i_x + \frac{v_a - v_b}{R_1} + \frac{v_a - v_c}{R_5} = 0$$

$$\textcircled{b} \quad -i_y + i_z + \frac{v_b - v_a}{R_1} = 0$$

$$\textcircled{c} \quad +i_y + \frac{v_b}{R_4} + \frac{v_c - v_a}{R_5} = 0$$

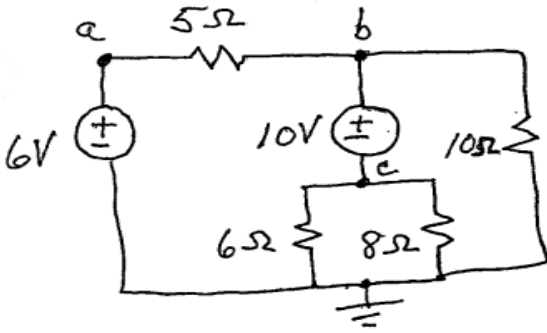
Exercise



Find three linear independent equations that you would produce to solve for V_a , V_b , V_c (the three unknowns) when using the node method. Explain the basic circuit laws you used to produce each equation.

Exercise

Solve for the node voltages. Verify your answers.

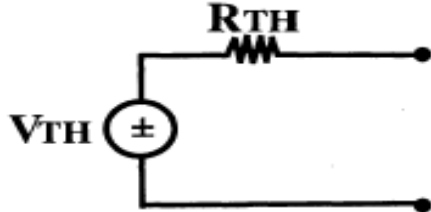


$$v_a = \boxed{} \quad v_b = \boxed{}$$

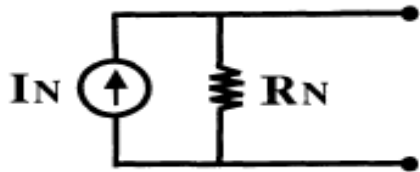
$$v_c = \boxed{}$$

Thevenin and Norton Equivalents

The Thevenin circuit is a linear circuit that has one resistor and one voltage source:



The Norton circuit is a linear circuit using one resistor and one current source:



We often simplify linear circuits to their Thevenin or Norton equivalents (the Thevenin or Norton circuit with the identical IV characteristic).

Exercise: Show that the same equivalent resistance R_{EQ} works for both Thevenin and Norton:

$$\boxed{R_{EQ} = R_{TH} = R_N}$$

$$\boxed{V_{TH} = I_N * R_N}$$

Computing R_{EQ}

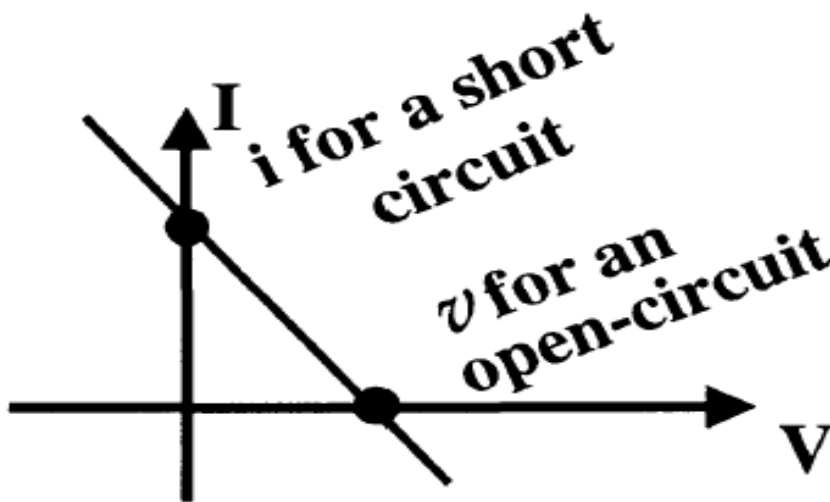
Compute the open-circuit voltage, the short-circuit current, and use ohm's law:

$$R_{EQ} = |V_{OC} / I_{SC}|$$



Be sure you understand why this works. Be sure you use the proper direction of current and voltage polarity.

Be sure you know how to read V_{OC} and I_{SC} on an IV characteristic:

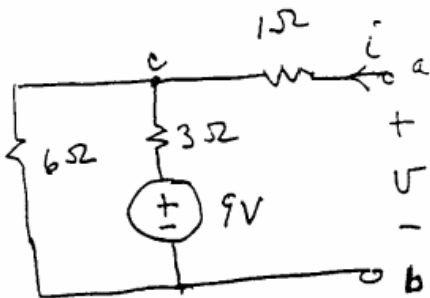


Shortcut for Computing R_{EQ} : Compute R_{EQ} with voltage sources shorted, current sources open.

Exercise 1

Find V_{OC} , I_{SC} , and R_{EQ} two ways:

- (1) Using R_{EQ} shortcut (redraw)**
- (2) Without shortcut, using Thevenin-Norton conversions.**

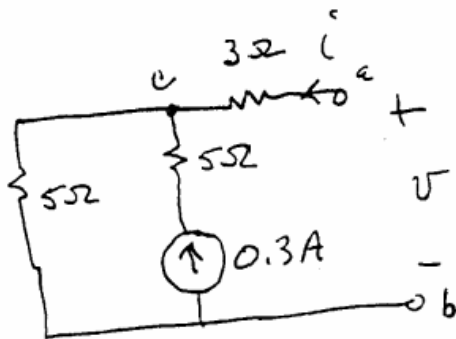


Exercise 2

Find V_{OC} , I_{SC} , and R_{EQ} any way you can.

Here are possible three ways you might do it:

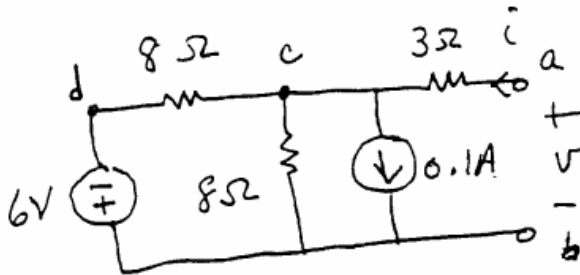
- (1) Use R_{EQ} shortcut, then find V_{OC} or I_{SC} (whichever is easier), then remaining values.
- (2) Use Node Voltage method at node C.
- (3) Use current divider equation.



Exercise 3

Find V_{OC} , I_{SC} , and R_{EQ} two ways:

- (3) Using R_{EQ} shortcut, then superposition to find V_{OC} , then I_{SC} .
- (4) Without shortcut, using Thevenin-Norton conversions.



Quiz LH16: Find the Thevenin and Norton equivalent circuits for the circuits below. Draw the I-V curve for each one.

