ECE 190 - Homework 1 Solutions

2.3.
a. $\lceil \log_2 400 \rceil = 9$ bits.
b. With 9 bits, we have $2^9 = 512$ patterns, so $512 - 400 = 112$ more students can be admitted before additional bits are required.

2.9. The range of an n-bit 2's complement data type is -2^{n-1} to $2^{n-1} - 1$. For Avogadro's number, we thus need $\lceil \log_2(6.02 \times 10^{23} + 1) \rceil + 1 = 80$ bits.

2.10.
a. $\overline{1010}$ = 0101
0101 + 1 = 0110
-$(0110)_2$ = $(-6)_{10}$

b. $(01011010)_2 = 2^1 + 2^3 + 2^4 + 2^6 = (90)_{10}$

c. $\overline{11111110}$ = 00000001
00000001 + 1 = 00000010
-$(00000010)_2$ = $(-2)_{10}$

d. $(0011100111010011)_2 = 1 + 2 + 16 + 64 + 128 + 256 + 2048 + 4096 + 8192 = (14803)_{10}$

2.11.
a. 102 = 01100110
b. 64 = 01000000
c. 33 = 00100001
d. -128 = 10000000
e. 127 = 01111111

2.12. Saying that a number is even is equivalent to saying that it is a multiple of two. Similarly, if the last two digits of a 2's complement number are 00, the number is a multiple of four.

2.18.
a. 01 + 1011 = 0001 + 1011 = 1100 = 12
b. 11 + 01010101 = 00000011+01010101 = 01011000 = 88
c. 0101+110=0101+0110=1011=11 (decimal)
d. 01+10=11=3

2.20. Parts (c) and (d) overflow.
a. -4 + 3 = 1100+0011 = 1111 = -1 (no overflow)
b. -4 + 4 = 1100+0100 = 0000 = 0 (no overflow)
c. 7+1 = 0111+0001=1000 = -8 (overflow)
d. -8 - 1 = 1000-0001=0111 = 7 (overflow)
e. 7+ -7 = 0111+1001=0000 = 0 (no overflow)

2.26. Again, the range of an n-bit 2's complement data type is -2^{n-1} to $2^{n-1} - 1$. The range of an n-bit unsigned data type is 0 to $2^n - 1$.
a. $\lceil \log_2 64 \rceil + 1 = 7$ bits
b. $2^6 - 1 = 63 = 0111111$
c. $2^7 - 1 = 127 = 1111111$

2.34.

a. NOT(1011) OR NOT(1100) = 0100 OR 0011 = 0111
b. NOT(1000 AND (1100 OR 0101)) = NOT(1000 AND 1101) = NOT(1000)=0111
c. NOT(NOT(1101)) = NOT (0010) = 1101
d. (0110 OR 0000) AND 1111 = 0110 AND 1111 = 0110
Note : Any number 'OR'ed with 0000 is the same number and any number 'AND'ed with 1111 is the same number

2.48.

a. 256 = x100 (10-bit or 12-bit 2's complement)
b. 111 = x6F (8-bit 2's complement)
c. 123,456,789 = x75BCD15 (28-bit 2's complement)
d. -44 = x54 (7-bit 2's complement, or xD4 in 8-bit 2's complement)

2.49

a. x025B = $(0000\,0010\,0101\,1011)_2 = (603)_{10}$
x26DE = $(0010\,0110\,1101\,1110)_2 = (9950)_{10}$
$\Rightarrow 0000\,0010\,0101\,1011 + 0010\,0110\,1101\,1110 = 0010\,1001\,0011\,1001 = (10553)_{10}$

b. x7D96 = $(0111\,1101\,1001\,0110)_2 = (32150)_{10}$
x26DE = $(0010\,0110\,1101\,1110)_2 = (9950)_{10}$
$\Rightarrow 0111\,1101\,1001\,0110 + 0010\,0110\,1101\,1110 = 1010\,0100\,0111\,0100 = (42100)_{10}$

c. xA397 = $(1010\,0011\,1001\,0111)_2 = (-23656)_{10}$
xA35D = $(1010\,0011\,0101\,1101)_2 = (-23727)_{10}$
$\Rightarrow 1010\,0011\,1001\,0111 + 1010\,0011\,0101\,1101 = 0100\,0110\,1111\,0100 = (18164)_{10}$ (overflow)

d. x7D96 = $(0111\,1101\,1001\,0110)_2 = (32150)_{10}$
x7412 = $(0111\,0100\,0001\,0010)_2 = (29714)_{10}$
$\Rightarrow 0111\,1101\,1001\,0110 + 0111\,0100\,0001\,0010 = 1111\,0001\,1010\,1000 = (-3672)_{10}$ (overflow)

2.54.

X	Y	Z	Q_1	Q_2
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0