

ECE 413: Problem Set 6

- Due:** Wednesday **October 11** at the beginning of class.
Reading: Ross, Chapters 3 and 4
Reminder: **HOURLY EXAM I: Monday October 9**
7:00 p.m. – 8:00 p.m., 119 MSB
 One $8\frac{1}{2}$ " \times 11" sheet of notes permitted
 Coverage: through Problem Set 5 and lecture of October 4
Noncredit Exercises: **Chapter 3:** Problems 29, 30, 35-38, 42, 43, 48-50

This Problem Set contains five problems

- In Problem 6 of Problem Set 2, we found that the House of Commons Search Committee randomly selects one of 20 short lists of three candidates from the set {Andy, Beth, Chuck, Di, Eddie, Fergie}. Before the House of Commons selects the new monarch, however, Di passes away. Now, due to partisan politics, however, the House will always choose Beth as monarch if she is on the short list. If Beth is not on the short list, then the House chooses at random from the three (only two if the list originally included Di) on the list.
 - What is the probability that Beth is chosen as the monarch?
 - What is the probability that Chuck is chosen as the monarch?
 - Given that Beth was chosen as the monarch, what is the conditional probability that Di was originally on the short list? that Chuck was on the short list?
- ["Take me out to the ball game. . ."] A baseball pitcher's repertoire is limited to *fastballs* (event F), *curve balls* (event C) and *sliders* (event S). It is known that $P(C) = 2P(F)$. Let H denote the event that the batter hits the ball. Whether H occurs or not depends on the pitch, and it is known that $P(H|F) = 2/5$, $P(H|C) = 1/4$, and $P(H|S) = 1/6$. If $P(H) = 1/4$, what is $P(C)$?
- We have two urns \mathcal{A} and \mathcal{B} each one containing identical red and blue balls. \mathcal{A} contains x blue and x red balls while \mathcal{B} contains $2x$ blue and x red balls where $x > 1$. A biased coin with probability of Heads p is tossed to select the urn to be used in an experiment. If the outcome is Heads, we use \mathcal{A} ; if it shows tails, then we use \mathcal{B} . We toss the coin to determine which urn is to be used; and then proceed as follows:
 - Randomly draw one ball from the urn to be used. What is the probability that it is a red ball? Given that the ball drawn is red, what is the probability that we are using urn \mathcal{B} ?
 - Randomly draw *another ball* from the urn *without replacing the first ball drawn*. What is the probability that both balls are red? What is the probability that both balls have the same color? Given that we have drawn two red balls, what is the probability that we are using urn \mathcal{A} ?
- Monty Hall, the host of the TV game show "Let's Make A Deal" shows you three curtains. One curtain conceals a car, while the other two conceal goats. All three curtains are equally likely to conceal the car. He offers you the following "deal": pick a curtain, and you can have whatever is behind it. When you pick a curtain, instead

of giving you your just deserts, Monty (who knows where the car is) opens one of the remaining curtains to show you that there is a goat behind it, and offers the following “new, improved deal” : you can either stick with your original choice, or switch to the remaining (unopened) curtain. Amidst the deafening roars of “Stand pat!” and “Switch, you idiot!” from the crowd, Monty points out that previously your chances of winning were $1/3$. Now, since you know that the car is behind one of the two unopened curtains, your chances of winning have increased to $1/2$, and thus the new improved deal is indeed better. Use the theorem of total probability to determine

- (a) the probability of winning if you always switch.
- (b) the probability of winning if you would rather fight than switch.
- (c) whether Monty is correct in asserting that if you choose randomly between the two unopened curtains, you have a probability of winning of $1/2$.
- (d) Having disposed of your goat, you return the next day to the show, and this time, Monty calls you *and* your friend to come on down and choose one curtain each. Which is better: to be the first to pick a curtain or the second? Or does it not make a difference? This time, Monty opens the curtain chosen by your friend to reveal a goat and sends him back to his seat. He now asks whether you want to stick with your original choice or switch to the the third (unchosen) curtain. Which choice gives you a larger chance of winning the car?

Note: Everybody knows that the rules of the game of parts (a)-(c) are that Monty always opens one of the two unchosen curtains and he always offers the “new improved deal,” i.e. he never opens a curtain to reveal the prize (saying “Oops, you lose; return to your seat.”). In the game of part (d), he always opens one of the chosen curtains to eliminate one of the contestants and then always offers the other contestant the chance to switch.

5. At the County Fair, you see a man sitting at a table and rapidly rolling a pea between three walnut shells. “Step right up, me bucko, and try your luck! The hand is quicker than the eye!” he says, and hides the pea under one of the shells. You have no idea which shell is covering the pea, but you point to one shell at random and bet that the pea is under it. The man picks up one of the shells that you didn’t choose, and shows you that the pea is not underneath that shell. He asks if you would like to switch your bet to the other unchosen shell. Should you accept the offer? Why or why not? How does this game differ from the one analyzed in Problem 4 parts (a)-(c)?