University of Illinois **ECE 313**
at Urbana-Champaign **Fall 2007**

Problem Set # 10

Assigned: Wednesday, October 31

Due: Wednesday, November 7

Coverage: This homework set covers the topics of functions of random variables (RVs); hazard rate functions; binary hypothesis testing for continuous RVs; and jointly distributed discrete RVs. The first topic was already covered by the previous homework set, but here you'll find a few more problems on it. The relevant sections of Ross for this problem set are: Chapter 5, Section 5.5.1 (pp. 234-236) and Section 5.7 (pp. 242-247), and Chapter 6, Sections 6.1-6.4 (pp. 258-291), except the continuous random variables parts. From the Lecture Notes on the class web-site, the relevant slides are Lectures 28-32.

In addition to the seven problems below (to be turned in), you are encouraged to attempt problems relevant to the topics above in the *Self-Test Problems and Exercises* section of Chapter 5 of Ross (pages 254-257), particularly 14, 15, and 16, and that of Chapter 6 of Ross (page 323), particularly 1, 2, and 4; their solutions can be found in Appendix B of Ross.

Note: This is the last homework set before the second midterm exam, scheduled for November 12, 2007.

PROBLEMS

60. X is a continuous random variable with pdf

$$f_X(x) = \begin{cases} c(1-x), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

where c is an appropriate constant. Let $Y = (1-X)^2$.

(a) Find the value of c.

(b) Obtain the cdf of Y.

(c) Obtain the pdf of Y.

61. X is a "Rayleigh random variable with parameter σ" if it has a pdf given by

$$f_X(x) = \begin{cases} \dfrac{x}{\sigma^2} \exp(-\dfrac{x^2}{2\sigma^2}), & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Let $Y = \min(X, E[X])$.

(a) Compute $E[X]$, the mean value of X.

(b) Obtain the complete expression for the cdf of Y. What type of a random variable is Y (continuous or discrete or mixed)?

(c) Compute $E[Y]$, either using the result of (b) above, or using LOTUS.

62. Each cell phone produced by a certain manufacturer is, independently, of acceptable quality with probability 0.95. What is the probability that at most 10 of the next 150 cell phones produced will be unacceptable? (Use the Gaussian approximation to the Binomial distribution, that is the De Moivre-Laplace theorem.)

63. The lung cancer hazard rate of a t-year-old male smoker, $\lambda(t)$, is such that

$$\lambda(t) = 0.027 + 0.00025(t - 40)^2 , \quad t \geq 40 .$$

Assuming that a 40-year-old male smoker survives all other hazards, what is the probability that he survives to **(a)** age 50, and **(b)** age 60, without contracting lung cancer?

64. If X has a hazard rate function $\lambda(t)$, compute the hazard rate function of $Y = aX$ where a is a positive constant. Verify the relationship you have obtained for the case where X is a uniform random variable over $(0, 1)$. That is compute the hazard rate functions of X and Y, and show that they satisfy the relationship that you have derived.

65. Consider the binary hypothesis testing problem where under hypothesis H_0, we have the pdf

$$f_0(x) = \begin{cases} c_0 x^2, & |x| \leq 1 \\ 0, & \text{else} \end{cases}$$

and under hypothesis H_1,

$$f_1(x) = \begin{cases} c_1(3 - |x|), & |x| \leq 3 \\ 0, & \text{else} \end{cases}$$

(a) Evaluate the constants c_0 and c_1.
(b) Find the ML decision rule, and compute the average probability of error under ML when the prior probability of H_0 is $\pi = 0.25$.
(c) Find the MAP decision rule when the prior probability of H_0 is as in (b) above. Clearly describe the decision regions, and compute the average probability of error under MAP.

66. Suppose that 3 balls are chosen without replacement from an urn containing 5 white and 8 red balls. Let X_i equal 1 if the ith ball selected is white, and let it equal 0 otherwise. Let $Y = X_1 X_2$ and $Z = X_1 X_2 X_3$.
(a) Obtain the joint probability mass function (pmf) of X_1, X_2.
(b) Obtain the joint pmf of X_1, X_2, X_3.
(c) Compute $E[Y]$ and $E[Z]$. Also compute $\text{var}(Z)$, the variance of Z.
(d) Using the results in (a) and (b) above, compute the conditional probability of $X_3 = 1$ given that $X_1 = 1$ and $X_2 = 0$.
(e) Now compute the conditional probability of $X_1 = 1$ and $X_2 = 0$ given that $X_3 = 1$.

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