ECE 313 Probability with Engineering Applications **Fall 2007**
Homework 11 **Due Nov 28, 2007**

67. Let X and Y be two independent exponential random variables with parameters λ_x and λ_y respectively.
(a) Find the pdfs of $\max(X, Y)$ and $\min(X, Y)$.
(b) Find the pdf of $X + Y$.
(c) Find the pdf of X conditional on $X + Y = a$.
(d) Find the pdf of X conditional on $X \geq b$.
(Hint: Consider the cases when $\lambda_x = \lambda_y$ and when $\lambda_x \neq \lambda_y$.)

68. Is the following function

$$F(x, y) = \begin{cases} 0, & x + y < 1 \\ 1, & x + y \geq 1 \end{cases}$$

a valid joint CDF? Prove your answer and show your work.

69. The random variables X and Y have joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty \\ 0, & \text{o.w.} \end{cases}$$

(a) Sketch the xy-plane with an indication of the region where $f_{X,Y}(x, y)$ is non-zero.
(b) Find the joint CDF of X and Y. Specify the value of $F_{X,Y}(x, y)$ for all x and y.
(c) Find the marginal CDFs of X and Y.
(d) Find the marginal probability density functions of X and Y by:
(d1) Integrating the joint probability density functions; and
(d2) Differentiating the marginal CDFs found in part (c).
(e) Find $P(Y > 3X)$.
(f) For $\alpha > 0$, find $P(X + Y \leq \alpha)$.
(g) Use the result in part (f) to determine the pdf of the random variable $Z = X + Y$.

70. Let $f_{X,Y,Z}(x, y, z) = g(x, y, z)$ be the joint pdf of three continuous random variable X, Y, Z, which is symmetric with respect to its arguments (x,y,z), that is

$$g(x, y, z) = g(x, z, y) = g(y, x, z) = g(y, z, x) = g(z, x, y) = g(z, y, x)$$

Compute $P(X < Y < Z)$.
Hint: You are expected to obtain a numerical value, by using only the symmetry of g, not any other specific structure of g (which is not given).

71. Suppose the random point (X, Y) is uniformly distributed over the square region with corners at the points: (1,0), (0,1), (-1,0), and (0,-1).
(a) Calculate the marginal pdfs of X and Y. Are X and Y independent?
(b) Compute E[X] and Var(X).
(c) Calculate the pdf of $Z = X + Y$.