University of Illinois **ECE 313**
at Urbana-Champaign **Fall 2007**

Problem Set # 2

Assigned: Wednesday, August 30
Due: Wednesday, September 5
Coverage: This homework set is based on material covered in Chapter 2 of Ross.

In addition to the eight problems below (to be turned in), you are encouraged to attempt problems in the *Self-Test Problems and Exercises* section of Chapter 2 of Ross (pages 63-65), particularly 4, 6, 8, 13 and 17; their solutions can be found in Appendix B of Ross.

PROBLEMS

7. In an experiment with sample space Ω, let A, B, C and D be events with probabilities

$$P(A) = \frac{3}{8}\,,\ P(C) = \frac{1}{2}\,,\ P(A \cup B) = \frac{5}{8}\,,\ P(C \cap D) = \frac{1}{3}\,.$$

Furthermore, events A and B are disjoint, while events C and D satisfy the relationship $P(C \cap D) = P(C)P(D)$. Compute the following probabilities:

(a) $P(A \cap B)$ (b) $P(B)$ (c) $P(A \cap B^c)$
(d) $P(A \cup B^c)$ (e) $P(D)$ (f) $P(C \cup D)$
(g) $P(C \cap D^c)$ (h) $P(C \cup D^c)$ (i) $P(C^c \cap D^c)$

8. Poker dice is played by simultaneously rolling 5 dice. Obtain the following probabilities associated with this experiment:

(a) P(no two alike) (b) P(one pair)
(c) P(two pairs) (d) P(three alike)
(e) P(four alike) (f) P(all five alike)

9. Two symmetric dice have both had two of their sides painted red, two painted black, one painted yellow, and the remaining one painted white. When this pair of dice are flipped, what is the probability that both land on the same color?

10. An urn contains 3 red and 7 black balls. Alice and Ben withdraw balls from the urn consecutively until a red ball is selected. Find the probability that Alice is the one who selects the red ball.
(Alice draws the first ball, then Ben, then again Alice, and so on. There is no replacement of the balls drawn.)

11. A forest contains 20 elk, of which 5 are captured, tagged, and then released. A certain time later 4 of the 20 elk are captured. What is the probability that 2 of these 4 have been tagged?

12. Five people, designated as A, B, C, D, E, are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that

(a) there is exactly one person between A and B,

(b) there are exactly two people between A and B,

(c) there are three people between A and B?

13. For any sequence of events $E_1, E_2, \ldots$, define a new sequence $F_1, F_2, \ldots$ of disjoint events such that for all $n \geq 1$, the following relationship holds:

$$\cup_{i=1}^{n} F_i = \cup_{i=1}^{n} E_i$$

14. Let E and F be two events on a sample space Ω, with probabilities $P(E) = 0.9,\ P(F) = 0.8$.

(a) First show that $P(EF) \geq 0.7$.

(b) Then prove that for any two arbitrary events E and F, the following inequality holds:

$$P(EF) \geq P(E) + P(F) - 1$$

This is known as *Bonferroni's inequality.*

(c) Now generalize Bonferroni's inequality to n events, that is

$$P(E_1 E_2 \cdots E_n) \geq P(E_1) + \cdots + P(E_n) - (n - 1)$$

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