ECE 313 Probability with Engineering Applications **Fall 2007**
Homework 3 **Due Sept. 12, 2007**

15. For a nonnegative integer-valued random variable N, show that

$$E[N] = \sum_{n=1}^{\infty} P(\{N \geq n\}) \qquad \sum_{n=0}^{\infty} nP(\{N > n\}) = \frac{1}{2}(E[N^2] - E[N])$$

16. Let X is a random variable with pmf p_X.
(a) Suppose g is a one-to-one function. Show that

$$E[g(X)] = \sum_x g(x) p_X(x)$$

(b) Suppose now g is general and can be many-to-one. Show that (a) still holds.

17. Let X be a random variable having expected value μ and variance σ^2. Find the expected value and variance of

$$Y = \frac{X - \mu}{\sigma}$$

18. A gambling book recommends the following "winning strategy" for the game of roulette. It recommends that a gambler bet \$1 on red. If red appears (which has probability $\frac{18}{38}$, then the gambler should take her \$1 profit and quit. If the gambler loses this bet, she should make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote the gambler's winnings when she quits.
(a) Find $P(\{X > 0\})$.
(b) Are you convinced that the strategy is indeed a "winning" strategy?
(c) Find $E[X]$.

19. Let $n \geq 1$ be an integer and suppose the random variable X has the pmf

$$p_X(k) = \frac{2k}{n(n+1)}, \quad 1 \leq k \leq n$$

(a) Verify that p_X is a valid pmf.
(b) Let $Y = 1/X$. Describe the pmf of the random variable Y.
(c) Compute $E[Y]$.

20. Consider a random variable X with pmf of

$$p_X(k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

for some $\lambda \geq 0$ and nonnegative integers k.
(a) Show that the mass function sums to one.
(b) Express $E[X]$ in terms of λ.
(c) Express $E[X(X-1)]$ in terms of λ.
(d) Using results from (b) and (c), express $\mathrm{Var}(X)$ in terms of λ.
(e) Express $E[z^X]$ in terms of z and λ.