University of Illinois **ECE 313**
at Urbana-Champaign **Fall 2007**

Problem Set # 4R

Assigned: Wednesday, September 12
Due: Wednesday, September 19
Coverage: This homework set is based on material covered in Chp 4 of Ross (except Sect 4.7), and pp. 430-431. You are also encouraged to read Chp 8, Sects 8.1-8.3 (pp. 261-269) of Yates & Goodman, on reserve in the Engineering Library.

In addition to the seven problems below (to be turned in), you are encouraged to attempt problems in the *Self-Test Problems and Exercises* section of Chapter 4 of Ross (pages 201-204), particularly 8, 10, 18, 19, 20 and 21; their solutions can be found in Appendix B of Ross.

PROBLEMS

21. In some military courts, 9 judges are appointed. However, both the prosecution and the defense attorneys are entitled to a peremptory challenge of any judge, in which case that judge is removed from the case and is not replaced. A defendant is declared guilty if the majority of judges cast votes of guilty, and s/he is declared innocent otherwise. Suppose that when the defendant is, in fact, guilty, each judge will (independently) vote guilty with probability 0.7, whereas when the defendant is, in fact, innocent, this probability drops to 0.3.

(a) What is the probability that a guilty defendant is declared guilty when there are (i) 9, (ii) 8, and (iii) 7 judges?

(b) If the prosecution attorney does not exercise the right to a peremptory challenge of a judge and if the defense is limited to at most two such challenges, how many challenges should the defense attorney make if s/he is 60 percent certain that the client is guilty?

22. A purchaser of transistors buys them in lots of 20. It is his policy to randomly inspect 4 components from a lot and to accept the lot only if all 4 are non-defective. If each component in a lot is, independently, defective with probability 0.1, what proportion of lots is rejected?

How would the answer above change if the lots are instead of size 100, and the purchaser adopts still the same policy.

23. Seven people hold reservations for travel in a 4-passenger shuttle from Champaign to O'Hare. Assume that the number of passengers who actually show up to travel is modeled as a Binomial random variable with parameters $(7, \frac{1}{2})$. If more than 4 people show up, only the first 4 get to go, and the rest are left behind (assume that no two people show up at exactly the same time). What is the average number of passengers that are left behind?

24. In an experiment, a biased coin with probability of Heads equal to p is tossed until we observe Heads for the *second* time. Let N denote the number of tosses needed to complete the experiment. As an example, if the outcomes are T-T-H-T-T-T-H, then $N = 7$.

(a) What are the values that N can take?

(b) Derive the probability mass function (pmf) of N.

(c) Given that $N = n$, what is the *Maximum Likelihood* estimate of p?
(The answer should be in terms of n.)

25. You are trying to persuade an empty-headed king that you can see the future. You offer to forecast what happens on repeated independent tosses of a biased coin of the realm which you happen to know has $P(\text{Heads}) = 0.11$.

(a) The skeptical king asks you to predict the number of Heads that will occur on the next 1000 tosses and promises to execute you if your guess is wrong, just to make it more interesting. Which number should you predict and why?

What is the probability that the 1000 coin tosses *do* result in the number of Heads you predicted?

(b) Suppose that you luck out and guess right in part (a). The next day, the king asks you to predict how many tosses will be required to observe the next Head. Which number should you predict and why?

What is the probability that a Head *does* occur for the first time on the toss you predicted?

(c) Since you guessed right twice in a row, the king is thinking that you can indeed see into the future, and assigns a harder problem: predict the number of tosses required to observe a Head for the 105th time. Which number should you predict and why?

What is the probability that a Head *does* occur for the 105th time on the toss you predicted?

26. Let X be a Binomial random variable with expected value $E[X] = 6$ and variance $\text{Var}(X) = 2.4$. Compute the following quantities:

(a) $P(X = 5)$ **(b)** $P(2X \geq 5)$ **(c)** $\text{Var}\,(3X - 2)$ **(d)** $E[X^3]$

27. In a simple experiment, a certain event A occurs with probability $P(A) = 0.01$. Consider a composite experiment which consists of repeated trials of the simple experiment. Let R_n be the relative frequency of occurrence of event A in the composite experiment when it consists of n independent trials of the simple experiment. How many trials n are needed so that the probability of R_n differing from $P(A)$ by more than 0.001 is less than 0.01?

Hint: Utilize the notion of *confidence interval*, and make use of the Chebyshev's inequality.

◇ ◇ ◇