ECE 313 Probability with Engineering Applications **Fall 2007**
Homework 5 **Due Sept. 26, 2007**

28. Consider an experiment of coin flips where each flip is a head with probability p and the flips are independent. Let $X_i = 1$ if the ith flip is a head and $X_i = 0$ otherwise.
(a) Suppose $p = 0.25$. Plot the pmf of the random variable $Y = 1/n \sum_{i=1}^{n} X_i$ for $n = 5, 10, 50, 200$. Be sure you scale the axes appropriately so that it is meaningful to compare the different plots.
(b) You want to estimate Y by specifying an interval symmetric around $E(Y)$ such that you can be 99% sure that Y will lie in that interval. Numerically calculate this interval for $p = 0.25$ and $n = 5, 10, 50, 200$. How does the size of this interval change with n? What do you think will happen as $n \to \infty$?
(c) For $n = 200$, calculate the size of the interval in (b) for different values of p, and plot the interval size against the variance of Y. Can you give an intuitive explanation for the trends in your graph?

29. (a) We are given a sample space Ω and a probability law P. Let B be an event with non-zero probability. Define the real-valued function Q on the set of events in Ω by $Q(A) := P(A|B)$. Show that Q satisfies the three axioms of a probability law and hence is a valid probability law.
(b) Using (a) or by direct calculations, show the the following formula:

$$P(A|B, C) = \frac{P(C|A, B)\, P(A|B)}{P(C|B)}$$

30. A laboratory blood test is 95 percent effective in detecting a certain disease when it is in fact present. However, the test also yields a false positive result for 1 percent of the healthy person tested. (That is, if a healthy person is tested, then with probability 0.01, the test will imply that he has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive? Are you surprised by the answer? Explain.

31. Lets consider the example in class where the document has n letters and the probability that the kth letter is typed wrongly is p.
(a) Explain why there is not enough information to calculate the probability that the entire document is typed correctly.
(b) Find the smallest possible value for this probability in terms of p and specify the probability law for which this smallest value is achieved.
(c) Let $n = 2$. Suppose you are told additionally that q_1 is the conditional probability that the second letter is typed wrongly given that the first letter is typed wrongly, and q_2 be the conditional probability that the second letter is typed wrongly given that the first letter is typed correctly. Do we now have a complete specification of the probability law?
(d) What constraint does q_1 and q_2 have to satisfy?
(e) Under what values of q_1 and q_2 is the probability that the document is typed correctly (i) equal to, (ii) larger than, and (iii) smaller than the probability when the events that the two letters are typed incorrectly are independent?

32. There are 2 machines having lifetimes distributed with pmf's p_1 and p_2. Suppose one of the 2 machines is randomly picked with equal probability and put in operation at time 0. Conditional on the fact that the machine is still running at time , what is the probability that it is machine 1 that was picked?