ECE 313 Probability with Engineering Applications **Fall 2007**
Homework 7 **Due Oct 17, 2007**

40. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2), & -1 < x < 1 \\ 0, & \text{o.w.} \end{cases}$$

(a) What is the value of c?
(b) What is the cumulative distribution function of X.

41. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2}, & x > 10 \\ 0, & x \le 10 \end{cases}$$

(a) Find $P(X > 20)$.
(b) What is the cumulative distribution function of X?
(c) What is the probability that of 6 such types of devices at least 3 will function for at least 15 hours? What assumptions are you making?

42. There are two cards, one marked with payoff X and one with payoff $2X$, where X is a random variable uniformly distributed in $[0, 1000]$. You randomly pick a card and see the payoff. You have the option of keeping the payoff or switching to the other card. Determine the optimal strategy.

43. Time is divided into slots of length Δ seconds. Packet arrival in any given time slot happens with probability p_Δ which is proportional to the length Δ, and packet arrivals in different time slots are independent events.
(a) Find the pmf and cdf of N, the number of time slots that I have to wait for the first packet arrival.
(b) Find the cdf of the waiting time T and approximate it by a pdf for small Δ.

44. Scott's company delivers 1 million packages a day. The probability that a package will be damaged in the delivery process is 0.01. Assume that product damages occur independently. What is the approximate probability that more than 10200 packages will be damaged a day?

45. Let X be a random variable with cdf

$$F_X(b) = \begin{cases} 0, & b < 1 \\ (1/3)b, & 1 \le b < 2 \\ 1, & b \ge 2 \end{cases}$$

(i) Sketch this cdf and determine whether X is a discrete, continuous, or mixed random variable.
(ii) Evaluate the probability that $|X - 1| < 1$.
(iii) Evaluate the conditional probability of $|X - 1| < 1$ given that $1 < X \le 2$, that is $P(|X - 1| < 1 | 1 < X \le 2)$.