University of Illinois **ECE 313**
at Urbana-Champaign **Fall 2007**

Problem Set # 8

Assigned: Wednesday, October 17

Due: Wednesday, October 24

Coverage: This homework set covers the topics of continuous and mixed random random variables, their expected value, variance and higher moments, and some specific distributions such as Gamma, Weibull, Cauchy, and Beta (in addition to uniform and exponential). The relevant reading material from Ross is Sections 5.2, 5.3, 5.5 (except 5.5.1) and 5.6.

In addition to the seven problems below (to be turned in), you are encouraged to attempt problems in the *Self-Test Problems and Exercises* section of Chapter 5 of Ross (pages 254-257), particularly 1, 3, 6, 13 and 18; their solutions can be found in Appendix B of Ross.

PROBLEMS

46. Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} 2(e^{-3x} + e^{-6x}), & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

(a) First show that this is a valid pdf.

(b) Compute the mean value and the variance of X, that is $E[X]$ and $\text{var}(X)$.

(c) Evaluate the probability that $|X| < 2$, that is $P(|X| < 2)$.

47. Let X be a random variable with cdf as defined in Problem 45 of the previous set, that is

$$F_X(b) = \begin{cases} 0, & b < 1 \\ (1/3)b, & 1 \leq b < 2 \\ 1, & b \geq 2 \end{cases}$$

(a) Compute the mean value and the variance of X, that is $E[X]$ and $\text{var}(X)$.

(b) Evaluate $E[3X^2 - 4]$.

48. Let X be a continuous random variable with pdf

$$f_X(x) = ce^{-2|x|}, \quad -\infty < x < \infty$$

where c is a positive constant. Let $Y = 3X - 2$.

(a) Obtain the value of c so that this is a valid pdf.

(b) Compute the mean value and the variance of Y, that is $E[Y]$ and $\text{var}(Y)$.

(c) What is the conditional probability of $Y > 4$ given that $Y > 0$, that is $P(Y > 4|Y > 0)$.

(d) Now compute the two conditional probabilities: $P(Y \geq 4|Y \geq 0)$ and $P(Y < 4|Y \geq 0)$.

49. **(a)** A fire station is to be located along a road of length A, $0 < A < \infty$. If fires will occur at points uniformly chosen on $(0, A)$, where should the station be located so as to minimize the expected distance from the fire? That is, choose a so as to minimize the quantity $E[|X - a|]$ when X is uniformly distributed over $(0, A)$.

(b) Now suppose that the road is of infinite length—stretching from point 0 outward to ∞. If the distance of a fire from point 0 is exponentially distributed with rate λ, where should the fire station now be located? That is, we want to minimize $E[|X - a|]$ with respect to a when X is now an exponential random variable with parameter λ.

50. You have 5 light bulbs each having an independent and exponentially distributed life time (in hours) with parameter $\lambda = 10$. At 6 pm you turn on all of them, and leave the room. You come back at 9 pm and observe that all bulbs are still operating. You leave the room again, and return at 11 pm.

(a) What is the probability that all bulbs are still operating?

(b) What is the probability that only two bulbs are still operating?

(c) If you see that only two bulbs are still operating, what is the probability that they both would still be operating by midnight?

51. Let $F_1(b)$ be the cdf of an exponential random variable with parameter $\lambda = 2$ and $F_2(b)$ be the cdf of a Bernoulli random variable with parameter $p = 0.4$. Let X be a random variable with cdf given by

$$F_X(b) = 0.3F_1(b) + 0.7F_2(b)$$

(a) Show that $F_X(b)$ is indeed a valid cdf. Is X a continuous, discrete, or mixed random variable?

(b) Compute the mean value and the variance of X, that is $E[X]$ and $\text{var}(X)$.

(c) Compute the probabilities

$$P(0 \le X < 2), \quad .P(0 < X \le 2), \quad .P(1 \le X \le 2), \quad .P(1 < X \le 2 | X \le 2)$$

52. You are given the quadratic equation

$$4x^2 + 4\beta x + \beta + 2 = 0$$

where β is a random variable. Find the probability that this equation has two distinct real roots, when

(a) β is uniformly distributed over $[0, 5]$.

(b) β is an exponential random variable with parameter $\lambda = 2$.

(c) β is a Poisson random variable with parameter $\lambda = 2$.

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