

ECE 313 Probability with Engineering Applications
Homework 9

Fall 2007
Due Oct 31, 2007

53. The lifetimes of a wireless chip produced by a certain semiconductor foundry are normally distributed with mean 1.4×10^6 hours and standard deviation 3×10^5 hours. What is the approximate probability that a batch of 100 chips will contain at least 20 whose lifetimes are less than 1.8×10^6 hours?

54. Consider the following channel with transmitted symbol $X = 1$ and received signal Y :

$$Y = aX + Z$$

Suppose Z is a Gaussian random variable with mean 0 and variance σ_n^2 . We will make the following decision: the transmitted symbol is 1 when $Y > 0$ and -1 otherwise. Find the probability of error in using this decision rule.

55. Suppose you have access to a Gaussian random variable $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$. Explain how you can use X to generate another Gaussian random variable $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$.

56. The arrival of customers at a bank is modeled as a Poisson process with rate λ . Suppose that two customers arrived during the first hour. What is the probability that

- (a) both arrived during the first 20 minutes;
- (b) at least one arrived during the first 20 minutes.

57. Cars cross a certain point in the highway in accordance with a Poisson process with rate 3 per minute. If Dave blindly runs across the highway, then what is the probability that he will be uninjured if the amount of time that it takes him to cross the road is s seconds?

58. Let X be a random variable with cdf as defined in Problem 45 of HW 7, that is

$$F_X(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{3}x, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

- (a) Find the cdf of $Y = 3X^2 - 4$.
- (b) Compute $E[Y]$ from $F_Y(y)$.

59. Let X be a Gaussian random variable with mean μ and variance σ^2 .

- (a) Find the distribution of e^{sX} .
- (b) Find $M(s) = E[e^{sX}]$.
- (c) What are $M'(0)$ and $M''(0)$?