University of Illinois, Urbana–Champaign Fall 2007

ECE 313: Solutions to Homework 1

1. (a) Any person can sit any seats. The answer is 8!

   (b) First consider A and B as one person, arrange seating for 7 people without any restriction. Then permute A and B for every possible arrangement. The answer is 7!2!

   (c) Same as (b), pair one man and one woman as one person. Arrange seating for 4 virtual people first to get 4!. There are also 4! unique ways to pair a man and a woman, same as arrange 4 men to 4 seats. There are two alternative man and woman seating for every arrangement and pairing, man on left or man or right. Total the answer is 4!4!2.

   (d) Group 5 men as one person. We got 4!. Then permuation of 5 men is 5!. The answer is 4!5!.

   (e) Assume their marriage stays during the seating. First we arrange seating for couples, get 4!. Then, for each couple and each seating, there are 2 ways, man left or man right. Note that two women can sit together in this case. The answer is $4!2^4$.

2. If all outcomes are unique, total outcomes for two experiments are $\sum_{i=1}^{m} n_i$.

3. The question is the same as how many ways to pick up 2 people from 20? The answer is $20 \cdot 19/2$.

4. For each outcome, the last number is always 6 otherwise the game won't stop. All other numbers must not be 6 otherwise it could have stopped. So the sample space is

$$\Omega = \{(\underbrace{x, \ldots, x}_{y}, 6) | x = 1, 2, 3, 4, 5.\, y = 0, 1, 2, \ldots\}$$

   Note that Ω has coutably infinite elements. Event E_n includes the outcomes that $y \geq n - 1$.

$$E_n == \{(\underbrace{x, \ldots, x}_{y}, 6) | x = 1, 2, 3, 4, 5.\, y = n - 1, n, n + 1, \ldots\}$$

   Full credit will be given to answers for $y < n - 1$ if correct reasoning and understanding is shown.

5. Draw a V-diagram or K-diagram will help solve the problem.

   (a) Prove $EF \subseteq E \subseteq E \cup F$. $\forall \alpha \in EF, \alpha \in EF$. Hence, $EF \subseteq E. \forall \alpha \in E, \alpha \in E \cup F$, and hence $E \subseteq E \cup F$.

   (b) By contradiction, if $F^c \not\subset E^c, \exists \alpha \in F^c, \alpha \notin E^c$, then $\alpha \in E$. So not all outcomes in E is in F, i.e. $E \not\subset F$. Therefore $F^c \subset E^c$ must be true.

   (c) By distributivity, $F = FE \cup FE^c = F(E \cup E^c) = F\Omega = F$.

   (d) $E \cup F = E \cup (E \cup E^c)F = E \cup EF \cup E^cF = E \cup E^cF$.

6. Axiom 1: Since the number of times that E occurs must be small or equal to the total number of experiments, $n(E) \leq n$. On the other hand the number is always greater or equal to 0. So $\forall E \subseteq \Omega, 0 \leq f(\cdot) \leq 1$.

   Axiom 2: The sum of number of times that E occurs and E does not occur is the total number of experiments. $n(E) + n(E^c) = n$. So $\forall E \subseteq \Omega, f(\Omega) = f(E \cup E^c) = f(E) + f(E^c) = \frac{n(E)+n(E^c)}{n} = 1$.

   Axiom 3: For any set of mutually exclusive events $E_1, E_2, \ldots E_n$, the number of occurrences for all of them is equal to sum of the number of occurrences for each of them. $n(E_1 \cup E_2 \cup \cdots \cup E_n) = n(E_1) + \cdots + n(E_n)$. Therefore $f(\cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} f(E_i)$.