7. We could first draw the Karnaugh map for A and B. Since A and B are disjoint, $AB = \emptyset$. Hence, $\Pr(AB) = 0$. $\Pr(A \cup B) = 1 - \Pr(A^c B^c)$, then we have $\Pr(A^c B^c) = 1 - 5/8 = 3/8$. $\Pr(AB^c) = \Pr(A) - \Pr(AB) = 3/8$. So the table is

	A	A^c
B	0	1/4
B^c	3/8	3/8

In a similar way, we can find the map for C and D. Since C and D are independent, i.e. $\Pr(CD) = \Pr(C)\Pr(D)$. Then, $\Pr(D) = 2/3$. $\Pr(C^c D) = \Pr(D) - \Pr(CD) = 1/3$. $\Pr(CD^c) = \Pr(C) - \Pr(CD) = 1/6$. So the table is

	C	C^c
D	1/3	1/3
D^c	1/6	1/6

Note that events represented by cells are all disjoint. The sum of events in cells is the whole sample space. So the sum of the probabilities in the table is 1. To find the probability of the events in question is to find the right cell events and sum up their probabilities.

(a) 0.

(b) 1/4=0.25. $\Pr(B) = \Pr(AB) + \Pr(A^c B)$.

(c) 3/8=0.375.

(d) 3/4=0.75. $\Pr(A \cup B^c) = \Pr(AB) + \Pr(AB^c) + \Pr(A^c B^c)$.

(e) 2/3=0.6667.

(f) 5/6=0.8333. $\Pr(C \cup D) = \Pr(CD) + \Pr(CD^c) + \Pr(C^c D)$.

(g) 1/6=0.1667.

(h) 2/3=0.6667. $\Pr(C \cup D^c) = 1 - \Pr(DC^c)$.

(i) 1/6=0.1667.

8. Each die has 6 faces and the outcome of each die is independent to all other dice. So the sample space is $\Omega = \{\{x\}^5 | x = 1, 2, 3, 4, 5, 6.\}$. The size of the sample space is $6^5 = 7776$.

(a) Let A be the event that the outcome of all 5 dices are different. Total possible outcomes in A is 6!. Therefore the probability of no two alike is $6!/6^5 = 0.0926$.

(b) Let A_1 be the event that face 1 is the pair that shows up. All other dice should show a distinct number other than 1. The number of outcomes in A_1 is the product of all possible combinations of 2 dice that forms the pair and the ways that the remaining 3 dice show distinct faces and other than 1, i.e. $\binom{5}{2}5!/2!$. Note that for all faces $i = 1, 2, 3, 4, 5, 6$, events A_i are disjoint and the number of unique outcomes in A_is are the same. Therefore, total number of ways for one pair is $6\binom{5}{2}5!/2! = 3600$, divided by sample space size to get the probability 0.4630.

(c) In a similar way, we choose two faces first, and then two pairs. Note that pairs are interchangeable. The face of the last one should not be the same as the faces shown in pairs. Total number of ways for two pairs is $\binom{6}{2}\binom{5}{2}\binom{3}{2}4 = 1800$. The answer is 0.2315.

(d) We first choose a face, and then the faces of the remaining two should be distinct. Total number of ways for this event is $6\binom{5}{3}5 \cdot 4 = 1200$. The answer is 0.1543.

(e) We choose a face, find the set of 4 and choose a distinct face for the rest one. Total number of ways for this event is $6\binom{5}{4}5$. The answer is 0.0193.

(f) The answer is $6/6^5 = 0.0008$.

9. Let R_i, B_i, Y_i, W_i where $i = 1, 2$ be the event that die i lands on face Red, Black, Yellow and White respectively. These events are mutually disjoint. Therefore, the probability that two dice have same face is $\Pr(R_1R_2) + \Pr(B_1B_2) + \Pr(C_1C_2) + \Pr(D_1D_2)$. By independence, we have $\Pr(R_1R_2) = \Pr(R_1)\Pr(R_2)$ and so forth. Since two dice are the same, $\Pr(R_i) = \Pr(B_r) = 1/3, \Pr(Y_i) = \Pr(W_i) = 1/6$. The answer is 5/18=0.2778.

10. Let A_i be the event that Alice first catches the red ball at her ith try. Note that $i = 1, 2, 3, 4$ because Ben's 4th try must be a red ball if all previous 7 tries are all black balls. A_is are disjoint events so the probability that Alice gets the red ball is $\sum_{i=1}^{4} \Pr(A_i)$. $\Pr(A_1) = 3/10$. $\Pr(A_2) = \frac{7}{10}\frac{6}{9}\frac{3}{8} = 0.175$. $\Pr(A_3) = \frac{7}{10}\frac{6}{9}\frac{5}{8}\frac{4}{7}\frac{3}{6} = 0.0833$. $\Pr(A_4) = \frac{7}{10}\frac{6}{9}\frac{5}{8}\frac{4}{7}\frac{3}{6}\frac{2}{5}\frac{3}{4} = 0.025$. The answer is 0.5833.

11. After the first catch, 20 elk are divided into two groups, 5 tagged and 15 untagged. In the next catch of 4, the event we want is that 2 fall in the first group and the other 2 fall in the second group. The number of outcomes for this event is $\binom{5}{2}\binom{15}{2}$. There is a total of $\binom{20}{4}$ ways to catch 4 elk in a group of 20. So the probability is $70/323 = 0.2167$.

12. There is a total of 5! ways to arrange 5 persons so the sample space size is 5!.
    (a) $\binom{3}{1}$ ways to pick up the person in the middle of A and B. Then group A, B and that person in one and arrange seatings for 3, rendering 3! ways. After all, A and B can have two ways to sit for each arrangement. The answer is $\binom{3}{1}3 \cdot 2/5! = 0.3$.
    (b) Choose two people in the middle, arrange seatings for 2 and then permute the seatings of A and B, and two persons in the middle respectively. The answer is $\binom{3}{2}2 \cdot 2 \cdot 2/5! = 0.2$.
    (c) In this case, A and B must be seated on the side. It is equivalent to arrange seatings for B, C, and D and permute A and B. The answer is $3!2/5! = 0.1$.

13. Let $F_1 = E_1, F_2 = E_2 \setminus F_1, F_3 = E_3 \setminus F_1 \cup F_2, \ldots, F_n = E_n \setminus \cup_{i=1}^{n-1} F_i$. Show that all F_is are disjoint events. For any two events F_i and F_j, where $i < j$, $F_i \cap F_j = \emptyset$ since $F_j \subseteq E_j \setminus F_i$. By induction, show that $\cup_{i=1}^{n} F_i = \cup_{i=1}^{n} E_i$. The base case $n = 1$ is trivial. If $\cup_{i=1}^{k} F_i = \cup_{i=1}^{k} E_i$ is true,

$$\begin{aligned}
\cup_{i=1}^{k+1} F_i &= \cup_{i=1}^{k} F_i \cup F_{k+1} \\
&= \cup_{i=1}^{k} F_i \cup E_{k+1} \setminus \cup_{i=1}^{k} F_i \\
&= \cup_{i=1}^{k} F_i \cup E_{k+1} \\
&= \cup_{i=1}^{k+1} E_i
\end{aligned}$$

    Therefore, $\cup_{i=1}^{n} F_i = \cup_{i=1}^{n} E_i$ for all integers $n > 0$.

14. (a) Since $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(EF) \leq \Pr(\Omega) = 1$, we have $\Pr(EF) \geq \Pr(E) + \Pr(F) - 1 = 0.7$.
    (b) (a) also shows the proof.
    (c) In general,

$$\begin{aligned}
\Pr(E_1E_2\cdots E_n) &\geq \Pr(E_1) + \Pr(E_2E_3\cdots E_n) - 1 \\
&\geq \Pr(E_1) + \Pr(E_2) + \Pr(E_3E_4\cdots E_n) - 2 \\
&\cdots \\
&\geq \Pr(E_1) + \cdots + P(E_n) - (n-1)
\end{aligned}$$

    A formal proof could be shown by induction.