

Probability with Engineering Applications

Lecture Notes for ECE 313

Fall 1997

Dilip V. Sarwate
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

© 1997 by Dilip V. Sarwate

All rights reserved. No part of this manuscript may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise without the prior written permission of the author.

Preface

These lecture notes, intended as a *supplement* to the required textbook for ECE 313: S. Ross, *A First Course in Probability* (5th edition) Prentice-Hall 1997, are based on the material that I used to teach ECE 313 in Fall 1997. The lecture notes (in Portable Document Format) were made available to interested students via the course home page. Most students did print them out for ease in reading. Many even brought the notes to class to save themselves the trouble of frantically scribbling notes during classroom discussions.

Not everything in these lecture notes was covered in class in Fall 1997. In contrast, I was able to cover all the material in Spring 1997 when I used viewgraphs and did not spend as much time writing on the blackboard. I have embellished the lecture material with some comments and remarks as well as several exercises, all of which are intended to help students understand the concepts presented in class. In some instances, the comments remind students of key facts that make it clear how a particular statement made in class (and in the notes) follows from the previously presented results. The comments occasionally have a few details that were skipped in classroom discussion, or (horrors!) they sometimes suggest that students work out the details of some derivation all by themselves. Some of these comments (usually in paraphrased form) were made orally during the lectures, and their inclusion in the lecture notes saved students the trouble of scribbling down these pearls of wisdom.

Other information about ECE 313, including problem sets from the Fall 1997 and Spring 98 offerings of ECE 313, can be found on the ECE Department Web Server with URL <http://www.ece.uiuc.edu/~ece313/>

Dilip V. Sarwate
Urbana, Illinois
December 1997

ECE 313: Probability with Engineering Applications Syllabus

	Hours
Approaches to Probability	5
<p>Subjective approach, classical approach, relative frequency approach, modeling issues; the axiomatic approach. Consequences of the axioms and examples, use of Venn diagrams and Karnaugh maps, principle of inclusion and exclusion.</p>	
Conditional Probability	6
<p>Definition of conditional probability, theorem of total probability, Bayes' formula and its use. Bayes' rule for deciding among competing hypotheses, maximum-likelihood (ML) rule, Type I and Type II errors</p>	
Independence and Independent Trials	6
<p>Stochastic independence of two events, Independence of multiple events, Reliability of systems and networks, Independent experiments and trials,</p>	
Random Variables	12
<p>Definition, Cumulative Distribution Function of a random variable: Discrete and continuous random variables: Mean and variance: mean, mode and median as measures of location, Markov's inequality; variance, Chebyshev's inequality and variance as a measure of spread. Examples of discrete and continuous distributions: Function of a random variable: expectation of a function of a random variable. Conditional Distributions: Reliability and Hazard rates: Hypothesis testing: Maximum-likelihood estimation of parameters of distributions:</p>	
Many Random Variables	11
<p>Joint distributions, covariance and correlation coefficient, jointly Gaussian random variables. Sums of random variables, other functions of many random variables, conditional distributions. Linear regression</p>	
Limit Theorems	2
<p>Markov's Inequality, Chebyshev's Inequality, Weak law of large numbers and central limit theorem</p>	
Exams	
Total	44

Table of Contents

Preface.....	iii
Syllabus.....	iv
Table of Contents	v
Chapter I. What is Probability?.....	1
The Classical Approach.....	1
Events.....	2
Sampling With Replacement and Without Replacement.....	4
The Relative Frequency Approach	6
The Axiomatic Approach.....	8
Consequences of the Axioms.....	11
Examples.....	12
Chapter II. Conditional Probability.	17
Consistency with the Relative Frequency Approach.....	18
Product Rule.....	20
Theorem of Total Probability.....	21
Bayes' Formula.....	23
Decision Making under Uncertainty.....	24
Maximum-likelihood Decision Rule.....	25
Error Probabilities.....	27
Bayesian Decision Rules.....	29
Decision s involving Costs.....	33
Maximum-likelihood Estimation.....	20
Chapter III. Independence.	39
Independence of Two Events.....	39
Independence of More than Two Events	41
Reliability of Triple Modular Redundancy Circuits.....	42
Reliability of More General Systems.....	43
Independent Experiments.....	45
Repeated Independent Trials	47
Decision Making under Uncertainty – Revisited.....	55
Likelihood Ratio.....	55
Threshold Tests.....	56
Error Probabilities.....	59
Automatic Repeat Request Communication Systems	43
Chapter IV. Random Variables.....	65
Definition and Nomenclature.....	66
The Cumulative Distribution Function.....	69

	Discrete Random Variables and Their Probability Mass Functions.....	73
	Continuous Random Variables and Their Probability Density Functions.....	75
	Functions of Random Variables.....	85
	Expectation of a Random Variable.....	94
	Mean, Mode and Median	94
	LOTUS.....	97
	Variance	98
	Exponential and Gamma Random Variables.....	99
	Poisson Random Processes.....	101
	Gaussian Random Variables.....	104
	Hazard Rates.....	108
	Conditional Distributions and Densities.....	111
	Decision Rules	112
Chapter V.	Many Random Variables.....	117
	Joint Distributions.....	118
	Discrete Random Variables	120
	Joint Probability Mass Functions	120
	Independent Discrete Random Variables.....	123
	Jointly Continuous Random Variables.....	124
	Joint Probability Density Functions	125
	Marginal Distributions and Densities	129
	Functions of Random Variables	131
	Independent Random Variables	136
	Maxima and Minima.....	141
	Multiple Functions of Many Random Variables	143
	Expectation	148
	Generalized LOTUS.....	148
	Covariance and Correlation	149
	Conditional Distributions and Densities.....	151
	Jointly Gaussian Random Variables.....	154
Chapter VI.	Limit Theorems.	161
	Markov's Inequality.....	161
	The Chernoff Bound.....	162
	Chebyshev's Inequality and the The Weak Law of Large Numbers	163
	The Strong Law of Large Numbers	165
	The Central Limit Theorem	165
Appendix.	Differentiation of an Integral	167