ECE 361: Advanced Digital Communication

Lecture 11: Modeling the Wireline Channel: Intersymbol Interference

Introduction

We are now ready to begin communicating reliably over our first physical medium: the wireline channel. Wireline channels (telephone and cable lines) are readily modeled as linear time invariant (LTI) systems (their impulse response changes very slowly – usually across different seasons of the year). In this lecture, we will arrive at a simple discrete-time model of the wireline channel, taking into account both the sampling operation and the LTI waveform of the channel itself. The main feature of this model is that the previously transmitted symbols affect the current received symbol. This feature is called *inter-symbol interference* (ISI) and is the main new challenge that has to be dealt with in wireline channels, apart from the (by now familiar) additive Gaussian noise.

Wireline Media

A wire is a single, usually cylindrical, elongated strand of drawn metal. The primary metals used in the conducting wire are aluminium, copper, nickel and steel and various alloys therein. We are used to several wireline media in our day-to-day life. We enumerate below a few of the common ones, along with a brief description of their physical composition and, more importantly, their impulse response characteristics.

1. *Telephone wire*: This connects houses and local telephone exchange, typically using a pair of copper conducting wires. They were designed to carry human voice which are all well contained in under 10 kHz. Depending on the condition of the wire and the length (distance between the house and the local telephone exchange) the telephone wire can be thought of as a low pass filter with bandwidth of about 1 or 2 MHz.

2. *Ethernet wire*: This is typically a collection of *twisted pairs* of wires: a form of wiring in which two conductors are wound together for the purposes of canceling out electromagnetic interference from external sources and crosstalk from neighboring wires. Twisting wires decreases interference because the loop area between the wires (which determines the magnetic coupling into the signal) is reduced.

   The twist rate (usually defined in twists per meter) makes up part of the specification for a given type of cable. The greater the number of twists, the greater the attenuation of crosstalk. Further, the length of the wire decides how much the transmitted signal is attenuated; the longer the distance, the more the attenuation. There has been a standardization of these cable qualities and are typically denoted as "Cat`x`" cables: where `x` stands for the different versions. For instance, the state-of-the-art technology is the Cat6 cable (`x` $= 6$) with four pairs of copper wires twisted together. The maximum allowed length is about 90 meters and the impulse response can be thought of as a low pass filter with the bandwidth of about 250 MHz.

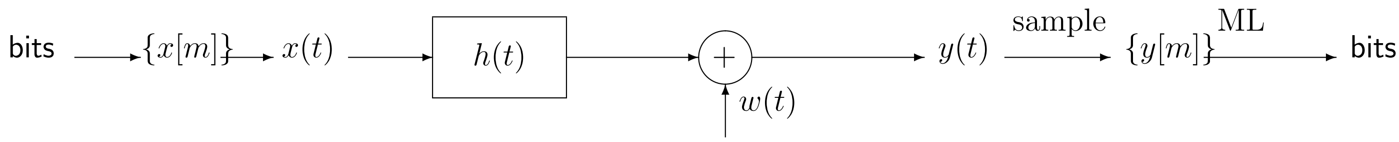

Figure 1: Illustration of the end-to-end digital communication process over a wireline channel.

3. *Cable TV wire*: This is typically a *coaxial* cable, consisting of a round conducting wire, surrounded by an insulating spacer, surrounded by a cylindrical conducting sheath, usually surrounded by a final insulating layer. Very high bandwidths are possible: for instance, RG-6, used for cable TV, has a bandwidth of about 500 MHz.

A Discrete Time Channel Model

A wire, as we have seen is simply a low pass filter; so, its effect can be completely captured in terms of its *impulse response*: $h(\cdot)$. Two important features of this impulse response are:

1. *Causality*: physical systems are causal and so

$$h(\tau) = 0, \quad \tau < 0. \tag{1}$$

2. *Dispersion*: This is the length time (denoted by T_d) over which a good large fraction of the total energy of the impulse response is contained within:

$$\int_0^{T_d} |h(\tau)|^2 \, d\tau \quad \approx \quad \int_0^{\infty} |h(\tau)|^2 \, d\tau. \tag{2}$$

If we transmit a voltage waveform $x(t)$ over a wire with impulse response $h(\cdot)$, then the received waveform is the *convolution* between $x(\cdot)$ and $h(\cdot)$ plus an additive noise waveform:

$$\int_0^{T_d} h(\tau)x(t-\tau) \quad d\tau \quad + \quad w(t). \tag{3}$$

In writing this equation, we have used the two properties of the wire enumerated above. We also denoted the additive noise waveform by $w(t)$.

From Lecture 8, we know that the transmit waveform $x(t)$ is formed by *pulse shaping* a discrete voltage sequence. Further, the received voltage waveform is sampled to generate a discrete sequence of (received) voltages (cf. Figure 1). In other words, the transmit waveform

$$x(t) = \sum_{m>0} x[m] g(t - mT) \tag{4}$$

where $g(\cdot)$ is the pulse shaping waveform (such as the raised cosine pulse). The received waveform is sampled at regular intervals (also spaced T apart, the same spacing as in the transmit waveform generation):

$$\begin{aligned} y[k] &\stackrel{\text{def}}{=} y(kT) && (5)\\ &= \int_0^{T_d} h(\tau) \left(\sum_{m>0} x[m] g(kT - mT - \tau) \right) \quad d\tau \quad + \quad w(kT) && (6)\\ &= \sum_{m>0} x[m] \left(\int_0^{T_d} h(\tau) g\left((k-m)\, T - \tau \right) \quad d\tau \right) + w[k]. && (7) \end{aligned}$$

We used Equations (3) and (4) in arriving at Equation (6). Further, we have denoted the noise voltage $w(kT)$ by $w[k]$ in Equation (7).

Denoting

$$h_{k-m} \stackrel{\text{def}}{=} \int_0^{T_d} h(\tau) g\left((k-m)\, T - \tau \right) \quad d\tau, \tag{8}$$

we can rewrite Equation (7) as

$$y[k] = \sum_{m>0} x[m] h_{k-m} + w[k], \quad k \geq 1. \tag{9}$$

Observing Equation (8) more closely, we see the following:

- Since the pulse $g(\cdot)$ has a finite *spread* of T_p, i.e. it is almost zero except for a period of time T_p (cf. the discussion in Lecture 8):

  $$g(t) \approx 0, \quad \forall t \notin (-t_0, T_p - t_0) \tag{10}$$

  for some t_0, we see that h_{k-m} is significantly non-zero for only a *finite* number of index values $(k - m)$.

- Specifically,

  $$h_{k-m} \approx 0, \quad (k-m)T \notin [-t_0, T_p + T_d - t_0]. \tag{11}$$

  Substituting $\ell = k - m$, we can say,

  $$h_\ell \approx 0, \quad \forall \ell \notin \left\{ -\left\lceil \frac{t_0}{T} \right\rceil, \ldots, \left\lfloor \frac{T_p + T_d - t_0}{T} \right\rfloor \right\}. \tag{12}$$

Substituting this observation in Equation (9), we can write the discrete received voltage *sequence* as

$$y[k] = \sum_{\ell = -\lceil \frac{t_0}{T} \rceil}^{\lfloor \frac{T_p + T_d - t_0}{T} \rfloor} h_\ell x[k - \ell] + w[k]. \tag{13}$$

So, the combination of transmit and receive processing (DAC and ADC) and the wireline channel effects the input-output relationship between the discrete voltage sequence in a very simple manner:

> the received voltage sequence is a *discrete convolution* of the transmit voltage sequence with an impulse response that has a *finite* number of nonzero coefficients along with the ubiquitous additive noise voltage sequence.

Inter-Symbol Interference

Viewed in the language of discrete time signal processing, the transmit voltage sequence has been passed through an FIR (finite impulse response) filter plus a noise voltage sequence. The number of non-zero coefficients of the channel (the number of *taps* in the FIR channel) is

$$L \stackrel{\text{def}}{=} \left\lceil \frac{T_p + T_d}{T} \right\rceil. \tag{14}$$

It is convenient to view *shifted* version of the received signal, shifted to the left by $-\left\lceil \frac{t_0}{T} \right\rceil$,

$$\tilde{y}[k] \stackrel{\text{def}}{=} y\left[k + \left\lfloor \frac{t_0}{T} \right\rfloor\right] \tag{15}$$

$$= \sum_{\ell=0}^{L-1} h_\ell x[k-\ell] + w\left[k - \left\lceil \frac{t_0}{T} \right\rceil\right]. \tag{16}$$

This way, the FIR filter involved is *causal* and makes for easier notation when we begin studying how to communicate reliably over this channel. As usual, we model the additive noise voltage sequence as Gaussian distributed (zero mean and variance σ^2) and all statistically independent from each other: this is the familiar *additive white Gaussian noise* model from the earlier lectures.

Henceforth, we will not bother to keep track of the shifted index in the received voltage sequence so explicitly in the main text. We will just write the received voltage sequence as

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell x[m-\ell] + w[m], \quad m \geq 1. \tag{17}$$

We can now see the main difference that the wireline channel makes as compared to the AWGN channel from before: the transmit voltages from a previous time sample also play a role in determining the received voltage at the present time sample. This phenomenon is known as *inter-symbol interference* or ISI for short: transmit voltages at different time samples (symbols) mix, or interfere, with each other to form the received voltages. The challenge of reliable wireline communication is to deal with the additive Gaussian noise in the presence of ISI.

Channel Coefficient Measurement

The channel coefficients $h_0, \ldots, h_{L-1}$ that make up the FIR filter depend on:

- sampling rate T;
- pulse shaping filter $g(\cdot)$;
- wireline channel impulse response $h(\cdot)$.

The first two quantities are chosen by the communication engineer (and hence known to both the transmitter and receiver). The final quantity depends on the physical characteristics of the wire involved. While the impulse response varies significantly from wire to wire, it is usually quite stable for a given wire (typically changing only over months along with the seasonal temperature changes). Of course, the time scale of practical reliable communication over the wire is far shorter. This means that the impulse response of the wire involved can be *learnt* at the very beginning part of the communication process and then used for reliable information transmission the rest of the time. In this section, we briefly describe this process of learning the channel coefficients.

Consider transmitting a voltage $+\sqrt{E}$ at the first time sample and nothing else after that. The received voltages of significance are:

$$y[\ell+1] = \sqrt{E}h_\ell + w[\ell+1], \quad \ell = 0, \ldots, L-1. \tag{18}$$

After these first L voltages, we only receive noise. Typical values of L are known ahead of time based on the wire involved (and the nature of the pulse shaping filter and sampling time). The precise number of taps L can also be learnt in this measurement phase; this is explored more in a homework exercise.

If $\sqrt{E} \gg \sigma$, then the received voltage $y[\ell+1]$ should provide a reasonably good estimation of h_ℓ. What is the appropriate scaling of $y[\ell+1]$ that yields a good estimate of h_ℓ? Suppose that we guess

$$\hat{h}_\ell \stackrel{\text{def}}{=} cy[\ell+1]. \tag{19}$$

Then the error in the estimate of h_ℓ is $(h_\ell - \hat{h}_\ell)$. A natural choice of the scaling constant c is to minimize the mean of the squared error:

$$\mathbb{E}\left[\left(h_\ell - \hat{h}_\ell\right)^2\right] = \mathbb{E}\left[(cy[\ell+1] - h_\ell)^2\right] \tag{20}$$

$$= \mathbb{E}\left[h_\ell^2\right]\left(1 - c\sqrt{E}\right)^2 + c^2\sigma^2, \tag{21}$$

since the channel coefficient h_ℓ and the noise voltage $w[\ell+1]$ are reasonably modeled as statistically independent of each other. Continuing from Equation (21), the variance of the error in the channel coefficient estimate is

$$\mathbb{E}\left[h_\ell^2\right]\left(1 - c\sqrt{E}\right)^2 + c^2\sigma^2 = c^2\left(\mathbb{E}\left[h_\ell^2\right]E + \sigma^2\right) - 2c\mathbb{E}\left[h_\ell^2\right]\sqrt{E} + \mathbb{E}\left[h_\ell^2\right]. \tag{22}$$

This quadratic function in c is readily minimized at

$$c^* \stackrel{\text{def}}{=} \frac{\mathbb{E}\left[h_\ell^2\right]\sqrt{E}}{\mathbb{E}\left[h_\ell^2\right]E + \sigma^2}. \tag{23}$$

To actually implement this scaling, we need to know the average attenuation of the ℓ^{th} channel coefficient. Typical values of such average attenuation are usually known based on the wire involved (that the telephone, ethernet and cable wires are *standardized*, allows for such data to be universally available). The corresponding error in the channel estimate is now

$$\begin{aligned} \mathbb{E}\left[h_\ell^2\right]\left(1-c^*\sqrt{E}\right)^2+\left(c^*\right)^2\sigma^2 &= \mathbb{E}\left[h_\ell^2\right]-\frac{\left(\mathbb{E}\left[h_\ell^2\right]\right)^2 E}{\mathbb{E}\left[h_\ell^2\right]E+\sigma^2} && (24)\\ &= \frac{\mathbb{E}\left[h_\ell^2\right]}{1+\mathbb{E}\left[h_\ell^2\right]\mathsf{SNR}}. && (25) \end{aligned}$$

Here we have denoted SNR as the ratio of E to σ^2. We can choose SNR to be large enough so that the variance of the error in the channel estimate is deemed small enough.

Looking Ahead

The wireline channel has now been simply modeled as an FIR filter acting on the transmit voltage sequence. Further, the coefficients of this filter are known before reliable data transmission is initiated (via the estimation process). We will see how to deal with the additive Gaussian noise in the presence of the ISI in the next several lectures.