ECE 361: Digital Communication

Lecture 18: Passband Wireless Communication

Introduction

Beginning with this lecture, we will study wireless communication. The focus of these lectures will be on point to point communication. Communication on a wireless channel is inherently different from that on a wireline channel. The main difference is that unlike wireline channel, wireless is a shared medium. The medium is considered as a federal resource and is federally regulated. The entire spectrum is split into many licensed and unlicensed bands. An example of the the point to point communication in the licensed band is the cellular phone communication, whereas wi-fi, cordless phones and blue tooth are some of the examples of communication in the unlicensed band.

The transmission over a wireless channel is restricted to a range of frequencies $\left[f_c - \frac{W}{2}, f_c + \frac{W}{2}\right]$ around the central carrier frequency f_c. Typically

$$f_c \gg W; \tag{1}$$

for example, $W \approx 1$ MHz and $f_c \approx 1$ GHz for cellular communication. On the other hand, the wireline channel is quite a contrast: the carrier frequency $f_c = 0$. Further more, the same wireless system (say cell phones) use different carrier frequencies in different cities. Clearly, it is not practical to tailor the communication strategies to the different carrier frequencies. It would be a lot better if we could design the system for a fixed carrier frequency and then have a simple mechanism to translate this design to suit the actual carrier frequency of operation.

This is indeed possible, and the fixed carrier frequency might as well be zero. This has the added advantage that it lets us borrow our earlier understanding of communication strategies on the wireline channel. In other words, the plan is to design for "baseband" even though we are looking to communicate in passband. The focus of this lecture is on this conversion. Finally we also see how the effects of the wireless channel (which is in passband, after all) translate to the baseband: in other words, we derive a "baseband equivalent" of the passband wireless channel.

Baseband Representation of Passband Signals

Let's begin with a baseband signal $x_b(t)$ (of double sided bandwidth W) that we want to transmit over the wireless channel in a band centered around f_c. From our discussion in the wireline channel, $x_b(t)$ would be the signal at the output of the DAC at the transmitter. We can "up convert" this signal by multiplying it by $\cos 2\pi f_c t$:

$$x(t) = \sqrt{2} \cos 2\pi f_c t \tag{2}$$

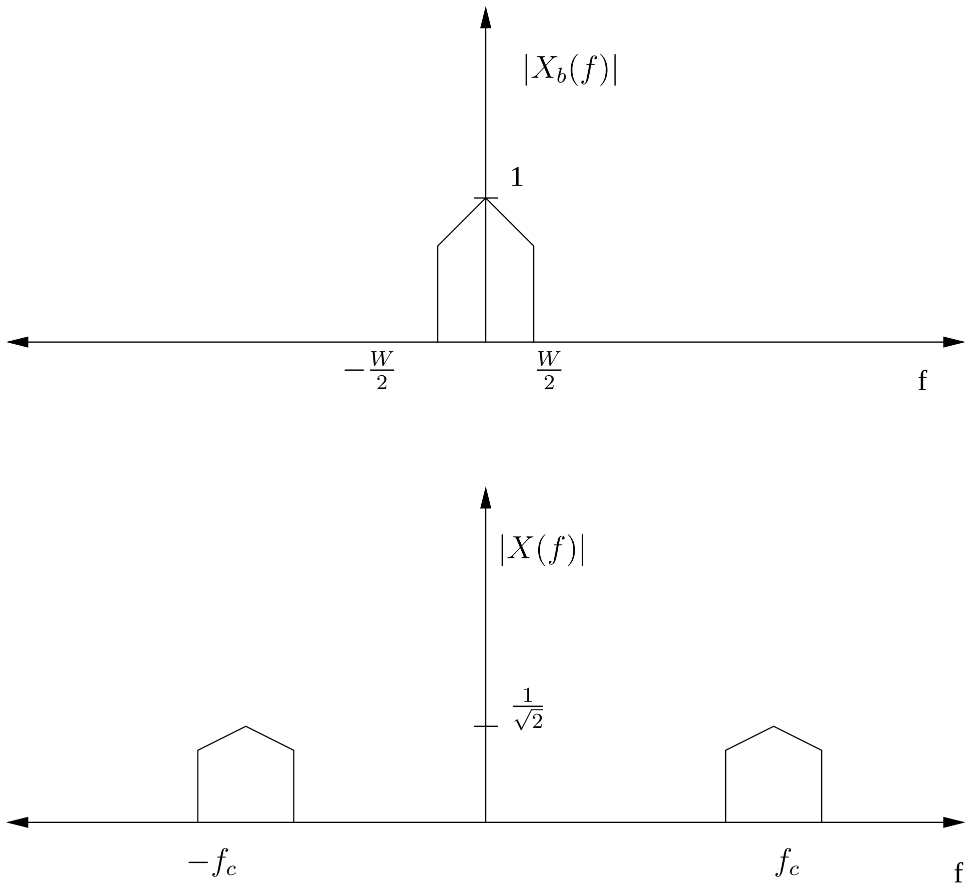

Figure 1: Magnitude spectrum of the real baseband signal and its passband signal.

is a passband signal, i.e., it's spectrum is centered around f_c and $-f_c$. Figure 1 shows this transformation diagrammatically. We scale the carrier by $\sqrt{2}$ as $\cos 2\pi f_c t$ has power $\frac{1}{2}$. Thus, by scaling, we are keeping the power in $x_b(t)$ and $x(t)$ same.

To get back the baseband signal, we multiply $x(t)$ again by $\sqrt{2}\cos 2\pi f_c t$ and then pass the signal through a low pass filter with bandwidth W:

$$x(t)\sqrt{2}\cos 2\pi f_c t = 2\cos^2(2\pi f_c t) x_b(t) \quad (3)$$

$$= (1 + \cos 4\pi f_c t) x_b(t) \quad (4)$$

The low pass filter will discard the signal $x_b(t)\cos 4\pi f_c t$ as it is a passband signal (centered around $2f_c$). Figure 2 shows this transformation diagrammatically.

One can see that if we multiply $x(t)$ by $\sqrt{2}\sin 2\pi f_c t$ instead of $\sqrt{2}\cos 2\pi f_c t$, we get $x_b(t)\sin 4\pi f_c t$ which would be eliminated completely by the low pass filter. There will be a similar outcome had we modulated the baseband signal on $\sqrt{2}\sin 2\pi f_c t$ and tried to recover it by using $\sqrt{2}\cos 2\pi f_c t$. This observation suggests the following engineering idea:

> We should upconvert two baseband signals, one using the cosine and the other using the sine and add them to make up the passband transmit signal. The

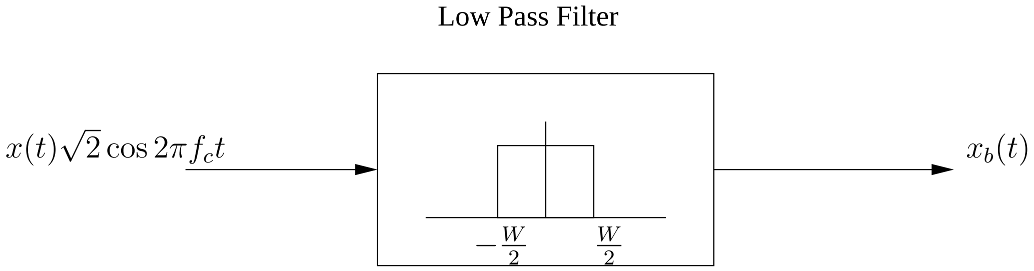

Figure 2: Down-conversion at the receiver.

receiver can recover each of the baseband signals by down converting by mixing with the cosine and sine waveforms in conjunction with a low pass filter.

Thus, we transmit *two* baseband signals in the same frequency band to create one passbnd signal. This is really possible because the total double sided bandwidth of a passband signal is $2W$ instead of just W in a baseband signal. To conclude: the passband signal $x(t)$ is created as

$$x(t) = x_{b_1}(t)\sqrt{2}\cos 2\pi f_c t - x_{b_2}(t)\sqrt{2}\sin 2\pi f_c t \tag{5}$$

The baseband signals $x_{b_1}(t)$ and $x_{b_2}(t)$ are obtained at the receiver by multiplying $x(t)$ by $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ separately and then passing both the outputs through low pass filters. We are modulating the *amplitude* of the carrier using the baseband signal and as such, this scheme is called *amplitude modulation*. When we use both the sin and cos parts of the carrier in the modulation process, the scheme is called *Quadrature Amplitude Modulation* (QAM). It is interesting to see if we can modulate a carrier using more than two independent baseband signals and yet recover each perfectly at the receiver. However this is not the case: from a basic trigonometric equality,

$$\cos(2\pi f_c t + \theta) = \cos\theta\cos 2\pi f_c t - \sin\theta\sin 2\pi f_c t. \tag{6}$$

We see that any modulation phase θ is uniquely determined by the amplitudes of $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$. Thus we conclude that a single passband signal is exactly represented by two baseband signals. The baseband signal $x_b(t)$ is now defined in terms of the pair

$$(x_{b_1}(t), x_{b_2}(t)). \tag{7}$$

In the literature, this pair is denoted as

$$(x_b^I(t), x_b^Q(t)), \tag{8}$$

where I stands for "in phase" signal and Q stands for "quadrature phase" signal. To make the notation compact we can think of $x_b(t)$ as a *complex* signal defined as follows:

$$x_b(t) \stackrel{\text{def}}{=} x_b^I(t) + j x_b^Q(t). \tag{9}$$

This allows us to represent passband signals by a single *complex* baseband signal. It also makes much of the material from our study of baseband communication (over the wireline channel) to be readily used in our passband scenario.

The Passband AWGN Channel

As in the wireline channel, the wireless channel is also well represented by a linear system. The main difference is that this channel is *time varying*, as opposed to the time-invariant nature of the wireline channel. For now, let us suppose that the wireless channel is time invariant. This will allow us to focus on the passband nature of the signals and channels first. We will return to the time variation at a later point in the lectures. Denoting by $h(t)$, the impulse response of the (time-invariant) wireless channel, the received passband signal is

$$y(t) = h(t) * x(t) + w(t). \tag{10}$$

Here $w(t)$ is passband noise. We can use the complex baseband representation derived earlier (in the context of the transmit passband signal $x(t)$) for the passband received signal $y(t)$ and the passband noise $w(t)$ (denoted, naturally, by $y_b(t)$ and $w_b(t)$ respectively). As can be expected, the passband signal equation in Equation (10) turns into the following baseband signal equation:

$$y_b(t) = h_b(t) * x_b(t) + w_b(t). \tag{11}$$

The key question is:

> How is the "baseband equivalent" $h_b(t)$ related to the passband channel $h(t)$?

The answer to this question will let us work directly with the baseband channel in Equation (11) which is almost the same as the one for the wireline channel (except that all signals are complex instead of real).

To understand the relation between $h(t)$ and $h_b(t)$, let us consider a few examples.

1. We start with a very simple scenario:

$$h(t) = \delta(t). \tag{12}$$

   Now

$$y(t) = x(t) + w(t) \tag{13}$$

   and hence

$$y_b(t) = x_b(t) + w_b(t). \tag{14}$$

   We conclude for this case that

$$h_b(t) = h(t) = \delta(t). \tag{15}$$

2. Now we move to a slightly more complicated scenario:

$$h(t) = \delta(t - t_0). \tag{16}$$

   Now

$$y(t) = x(t - t_0) = x_b^I(t - t_0)\sqrt{2}\cos 2\pi f_c(t - t_0) - x_b^Q(t - t_0)\sqrt{2}\sin 2\pi f_c(t - t_0). \tag{17}$$

The baseband signal $y_b(t)$ is composed of:

$$\begin{aligned}
y_b^I(t) &= \text{low pass filter output of } \left(y(t)\sqrt{2}\cos 2\pi f_c t \right) && (18)\\
&= \text{low pass filter output of } \big((2\cos 2\pi f_c(t-t_0)\cos 2\pi f_c t) x_b^I(t-t_0) \\
&\quad -(2\sin 2\pi f_c(t-t_0)\cos 2\pi f_c t) x_b^Q(t-t_0) \big) && (19)\\
&= \text{low pass filter output of } \big((\cos 2\pi f_c(2t-t_0) + \cos 2\pi f_c t_0) x_b^I(t-t_0) \\
&\quad -(\sin 2\pi f_c(2t-t_0) - \sin 2\pi f_c t_0) x_b^Q(t-t_0) \big) && (20)\\
&= x_b^I(t-t_0)\cos 2\pi f_c t_0 + x_b^Q(t-t_0)\sin 2\pi f_c t_0 && (21)\\
&= \Re\left\{ x_b(t-t_0) e^{-j2\pi f_c t_0} \right\}. && (22)
\end{aligned}$$

Similarly, we obtain $y_b^Q(t)$ as

$$\begin{aligned}
y_b^Q(t) &= \text{low pass filter output of } \left(-y_b(t)\sqrt{2}\sin 2\pi f_c t \right) && (23)\\
&= \Im\left\{ x_b(t-t_0) e^{-j2\pi f_c t_0} \right\}. && (24)
\end{aligned}$$

Thus,

$$y_b(t) = x_b(t-t_0) e^{-j2\pi f_c t_0}. \qquad (25)$$

We conclude that

$$\begin{aligned}
h_b(t) &= e^{-j2\pi f_c t_0}\delta(t-t_0) && (26)\\
&= e^{-j2\pi f_c t_0} h(t). && (27)
\end{aligned}$$

We notice that the baseband signal also gets delayed by the same amount as the passband signal. The key change is in the phase of the baseband signal (remember, it is a complex signal) depends on both the carrier frequency and tiem shift.

We are now ready to generalize: a general wireless channel is typically a sum of attenuated and delayed copies of the transmit signal. Specifically:

- the attenuation is due to the energy lost at the transmit and receive antennas, as well as any absorption in the medium;
- the delay is due to the time taken to travel the distance between the transmitter and receiver;
- the multiple copies are due to the fact that there are many physical paths between transmitter and receiver that are of the same order of attenuation.

In mathematical notation,

$$h(t) = \sum_i a_i \delta(t-\tau_i). \qquad (28)$$

The complex baseband equivalent of the channel is, as a natural generalization of the examples we did earlier,

$$h_b(t) = \sum_i a_l e^{-j2\pi f_c \tau_i}\delta(t-\tau_i). \qquad (29)$$

Looking Ahead

In this lecture we saw that passband communication can be represented readily by complex baseband communication. This representation is both in terms of the notation and mathematics, but also in terms of concrete engineering: we only need to upconvert the complex baseband signal at the transmitter and downconvert at the receiver. In the next lecture, we will add the usual DAC and ADC blocks and arrive at the discrete time complex baseband representation of passband communication. Finally, we will also be able to arrive at appropriate statistical models of the wireless channel.