ECE 361: Digital Communication

Lecture 19: The Discrete Time Complex Baseband Wireless Channel

Introduction

In the previous lecture we saw that even though the wireless communication is done via passband signals, most of the processing at the transmitter and the receiver happens on the (complex) baseband equivalent signal of the real passband signal. We saw how the baseband to passband conversion is done at the transmitter. We also studied simple examples of the wireless channel and related it to the equivalent channel in the baseband. The focus of this lecture is to develop a robust model for the wireless channel. We want the model to capture the essence of the wireless medium and yet be generic enough to be applicable in all kinds of surroundings.

A Simple model

Figure 1 shows the processing at the transmitter. We modulate two data streams to generate the sequence of complex baseband voltage points $x_b[m]$. The real and imaginary parts of $x_b[m]$ pass through the D/A converter to give baseband signal $x_b(t)$. Real and imaginary parts of $x_b(t)$ then modulates cos and sin parts of the carrier to generate the passband signal $x(t)$. The passband signal $x(t)$ is transmitted in the air and the signal $y(t)$ received.

Given all the details of the reflectors and absorbers in the surroundings, one can possibly use Maxwell's equations to determine the propagation of the electromagnetic signals and get $y(t)$ as an exact function of $x(t)$. However, such a detailed model is neither required nor is desired. The transmitter and receiver antennas are typically separated by several wavelengths apart and far field approximations of the signal propagation are good enough. Secondly, we do not want the model to be very specific to certain surrounding. We want the model to be applicable to most of the surroundings and still be meaningful.

We can model the electromagnetic signal as rays. As the rays travel in the air, they get attenuated. There is a nonzero propagation delay that each ray experiences. Further, the rays gets reflected by different reflectors before reaching the receiver. Thus, the signal arrives at the receiver via multiple paths, each of which sees different delay and attenuation. There is also an additive noise present at the receiver.

Hence, we can have a simple model for the received signal $y(t)$ as

$$y(t) = \sum_i a_i x(t - \tau_i) + w(t), \tag{1}$$

where a_i is the attenuation of the i^{th} path and τ_i is the delay it experiences. $w(t)$ denotes the additive noise.

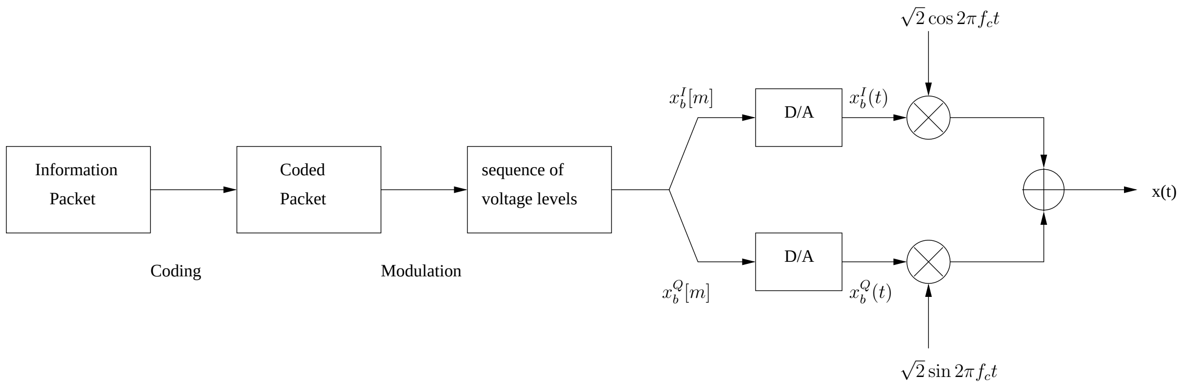

Figure 1: Diagrammatic representation of transmitter.

The delay τ_i is directly related to the distance traveled by the path i. If d_i is the distance traveled by the path i, then the delay is

$$\tau_i = \frac{d_i}{c} \tag{2}$$

where c is the speed of light in air. The typical distances traveled by the direct and reflected paths in the wireless scenario ranges from of the order of 10 m (in case of Wi-Fi) to 1000 m (in case of cellular phones). As $c = 10^8$ m/s, this implies that the delay values can range from 33 ns to 3.3 μs. The delay τ depends on the path length and is same for all the frequencies in the signal.

Another variable in Equation 1 is the attenuation a_i. In free space the attenuation is inversely proportional to the distance traveled by the path i, i.e., $a_i \propto \frac{1}{d_i}$. In the terrestrial communication, the attenuation depneds on the richness of the environment with respect to the scatterers. Depending on the environment, it can vary from $a_i \propto \frac{1}{d_i^2}$ to $a_i \propto e^{-d_i}$.

Scatterers can have different absorption coefficients for the different frequencies and the attenuation can depend on the frequency. However, we are communicating in a narrow band (in KHz) arround a high frequency carrier (in GHz). Thus, the variation within the band of interest are insignificant.

However, the most important aspect of the wireless communication is that the transmitter, the receiver and the surrounding are not stationary during the communication. Hence the number of path arriving at the receiver and the distance they travel (and hence the delay and the attenuation they experience) change with time. All these parameters are then functions of time. Hence, Equation 1 should be modified to incorporate this factor.

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t). \tag{3}$$

At the receiver $y(t)$ is down-converted to the baseband signal $y_b(t)$. Its real and imaginary parts are then sampled at the sampling rate W samples per second. Figure 2 depicts these operations.

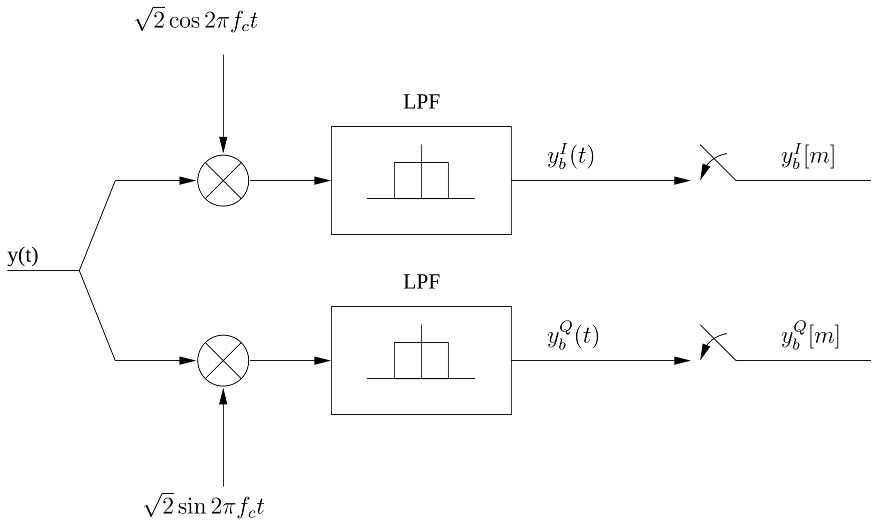

Figure 2: diagramatic representation of receiver

Discrete Time Channel model

Since the communication is in the discrete time instances, we want a model for the channel between $x_b[m]$ and $y_b[m]$. Let's try to obtain the discrete time baseband channel model from Equation 1. We keep in mind that the delays and the attenuation of the paths are time varying, though we will not explicitly write them as functions of time in the following discussion. Equation 1 can be written as

$$y(t) = h(t) * x(t) \tag{4}$$

where, $h(t)$ is the impulse response of the channel and is given by

$$h(t) = \sum_i a_i \delta(t - \tau_i). \tag{5}$$

From the previous lecture, we know the impulse response of the baseband channel $h_b(t)$ is given by

$$h_b(t) = \sum_i a_i e^{-j2\pi f_c \tau_i} \delta(t - \tau_i) \tag{6}$$

and the baseband received signal is

$$y_b(t) = h_b(t) * x_b(t) + w_b(t). \tag{7}$$

We know that $y_b[m]$ is obtained by sampling $y_b(t)$ with sampling interval $T = \frac{1}{W}$s.

$$\begin{aligned} y_b[m] &= y_b(mT) + w_b(mT) && (8) \\ &= \sum_i x_b(mT - \tau_i) a_i e^{-j2\pi f_c \tau_i} + w_b(mT) && (9) \end{aligned}$$

Recall that $x_b(t)$ is obtained from $x_b[n]$ by passing it through the pulse shaping filter. Assuming the ideal pulse shaping filter $\mathrm{sinc}(\frac{t}{T})$, $x_b(t)$ is

$$x_b(t) = \sum_n x[n]\mathrm{sinc}\left(\frac{t - nT}{T}\right). \tag{10}$$

Substituting in Equation 9, we get

$$\begin{aligned} y_b[m] &= \sum_n \sum_i x_b[n]\mathrm{sinc}\left(m - n - \frac{\tau_i}{T}\right) a_i e^{-j2\pi f_c \tau_i} + w_b[m] && (11) \\ &= \sum_n x_b[n] \left(\sum_i a_i e^{-j2\pi f_c \tau_i}\mathrm{sinc}\left(m - n - \frac{\tau_i}{T}\right)\right) + w_b[m]. && (12) \end{aligned}$$

Substituting $\ell := m - n$, we get

$$\begin{aligned} y_b[m] &= \sum_\ell x_b[m-\ell] \left(\sum_i a_i e^{-j2\pi f_c \tau_i}\mathrm{sinc}\left(\ell - \frac{\tau_i}{T}\right)\right) + w_b[m] && (13) \\ &= \sum_{\ell=0}^{L-1} h_\ell x_b[m-\ell] + w_b[m], && (14) \\ & && (15) \end{aligned}$$

where the tap coefficient h_ℓ is defined as

$$h_\ell \stackrel{\mathrm{def}}{=} \sum_i a_i e^{-j2\pi f_c \tau_i}\mathrm{sinc}\left(\ell - \frac{\tau_i}{T}.\right) \tag{16}$$

We recall that these are exactly the same calculations as for obtaining the tap coefficients for the wireline channel in Lecture 9. From Lecture 9, we recall that if T_p is the pulse width and T_d is the total delay spread, then the number of taps L are

$$L = \lfloor \frac{T_p + T_d}{T} \rfloor \tag{17}$$

where the delay spread T_d is the difference between the delays between the shortest and the longest path.

$$T_d \stackrel{\mathrm{def}}{=} \max_{i \neq j} |\tau_i - \tau_j|. \tag{18}$$

Note that Equation 14 also has the complex noise sample $w_b[m]$. It is the sampled baseband noise $w_b(t)$. We model the discrete noises $w_b[m]$, $m \geq 1$, as i.i.d. complex Gaussian random variables. Further we model both the real and imaginary parts of the complex noise as i.i.d. (real) Gaussian random variables with mean 0 and variance $\frac{N_0}{2}$.

Equation 14 is exactly the same as that of wireline channel equation. However, the wireline channel and wireless channel are not the same. We recall that the delays and attenuations of the paths are time varying and hence the tap coefficients are also time varying. We also note unlike in the wireline channel, both h_l and $x_b[m]$ are complex numbers.

The fact that the tap coefficients h_l and even the number of taps L are time varying is a distinguishing feature of the wireless channel. In wireline channel the taps do not change and hence they can be learned. But now we cannot learn them once and use that knowledge for rest of the communication. Further, the variations in the tap coefficients can be huge. It seems intuitive that the shortest paths will add up in the first tap and since these paths are not attenuated much, h_0 should always be a good tap. It turns out that this intuition is misleading. To see this, let's consider Equation 16. Note that the paths whose delays are separated by at most T seconds. For the tap h_0, $\tau_i \leq T$ and we can approximate $\mathrm{sinc}\left(-\frac{\tau_i}{T}\right) \approx 1$.

But note that the phase term $e^{-j2\pi f_c \tau_i}$ can vary a lot. The paths that have a phase lag π will have

$$f_c(\tau_1 - \tau_2) = \frac{1}{2} \tag{19}$$

$$\tau_1 - \tau_2 = \frac{1}{2f_c}. \tag{20}$$

With $f_c = 1$ GHz, $\tau_1 - \tau_2 = 0.5ns$. This corresponds to the difference in their path lengths to be of 15 cm. Thus, there could be many paths adding constructively and destructively and we could have a low operating SNR even when the transmitter and receiver are right next to each other.

We can now see that the key primary difference between the wire line and the wireless channels is in the magnitudes of the channel filter coefficients: in a wireline channel they are usually in a prespecified (standardized) range. In the wireless channel, however:

1. the channel coefficients can have a widely varying magnitude. Since there are multiple paths from the transmitter to the receiver, the overall channel at the receiver can still have a very small magnitude even though each of the individual paths are very strong.

2. the channel coefficients change in time as well. If the change is slow enough, relative to the sampling rate, then the overhead in learning them dynamically at the receiver is not much.

Even if the wireless channel can be tracked dynamically by the receiver, the communication engineer does not have any idea *a priori* what value to expect. This is important to know since the resource decisions (power and bandwidth) and certain traffic characteristics (data rate and latency) are fixed a priori by the communication engineer. Thus there is a need to develop *statistical* knowledge of the channel filter coefficients which the communication engineer can then use to make the judicial resource allocations to fit the desired performance metrics. This is the focus of the rest of this lecture.

Statistical Modeling of The Wireless Channel

At the outset, we get a feel for what statistical model to use by studying the reasons why the wireless channel varies with time. The principal reason is mobility, but it helps to separate the role as seen by different components that make up the overall discrete time baseband channel model.

1. The arrival phase of the constituent paths making up a single filter coefficient (i.e., these paths all arrive approximately within a single sample period) may change. The arrival phase of any path changes by π radians when the distance of the path changes by half a wavelength. If the relative velocity between the transmitter and receiver for path i is v_i, then the time to change the arrival phase by π radians is

$$\frac{c}{2f_c v_i} \quad \text{seconds.} \tag{21}$$

   Substituting for the velocity of light c (as 3×10^8 m/s) and sample carrier frequency f_c values (as 10^9 Hz) and gasoline powered velocity v_i (as 60 mph) we get the time of phase reversal to be about 5 ms.

2. Another possibility is that a new path enters the aggregation of paths already making up a given channel filter tap. This can happen if the path travels a distance less (or more) than previously by an order of the sampling period. Since sampling period T is inversely related to the bandwidth W of communication and the bandwidth is typically three orders or so less than the carrier frequency (say, $W = 10^6$ Hz), this event occurs over a time scale that is three orders of magnitude larger than that of phase change (so this change would occur at about 5 seconds, as opposed to the earlier calculation of 5 ms). As such, this is a less typical way (than the previous one) in which channel can change.

3. It could happen that the path magnitudes change with time. But this requires the distance between the transmitter and receiver to change by a factor of two or so. With gasoline powered velocities and outdoor distances of 1 mile or so, we need several seconds for this event to occur. Again, this is not the typical way channel would change.

In conclusion:

> The time scale of channel change is called *coherence interval* and is dominated by the effect of the phase of arrival of different paths that make up the individual channel filter tap coefficients.

How we expect the channel to change now depends on how many paths aggregate within a single sample period to form a single channel filter tap coefficient.

We review two popular scenarios below and arrive at the appropriate statistical model for each.

- **Rayleigh Fading Model**: When there are many paths of about the same energy in each of the sampling periods, we can use the central limit theorem (just as in Lecture 2) to arrive at a Gaussian approximation to the channel filter coefficients. Since the channel coefficient is a complex number, we need to arrive at a statistical model for both the real and imaginary parts. A common model is to suppose that both the real and imaginary parts are statistically independent and identically distributed to be Gaussian (typically with zero mean). The variance is proportional to the energy attenuation expected between the transmitter and receiver: it typically depends on the distance between the transmitter and receiver and on some gross topographical properties (such as indoors vs outdoors).

- **Rician Fading Model**: Sometimes one of the paths may be strongly dominant over the rest of the paths that aggregate to form a single channel filter coefficient. The dominant path could be a line of sight path between the transmitter and receiver while the weaker paths correspond to the ones that bounce off the objects in the immediate neighborhood. Now we can statistically model the channel filter coefficient as a Rayleigh fading with a non-zero mean. The stronger the dominant path relative to the aggregation of the weaker paths, the larger the ratio of the mean to the standard deviation of the Rayleigh fading model.

Looking Ahead

Starting next lecture we turn to using the statistical knowledge of the channel to communicate reliably over the wireless channel.