ECE 361: Fundamentals of Digital Communications

Lecture 23: Frequency Diversity

Introduction

In many instances, temporal diversity is either not available (in stationary scenarios) or cannot be efficiently harnessed due to the strict delay constraints of the data being communicated. In these instances, and as an added source of diversity, looking to the frequency domain is a natural option.

Frequency Diversity Channel

Frequency diversity occurs in channels where the multipaths are spread out far enough, relative to the sampling period, so that multiple copies of the same transmit symbol are received over *different* received samples. Basically, we want a multitap ISI channel response: the L-tap wireless channel

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell x[m-\ell] + w[m], \quad m \geq 1 \tag{1}$$

is said to have L-fold frequency diversity. The diversity option comes about because the different channel taps $h_0, \ldots, h_{L-1}$ are the result of different multipath combinations and are appropriately modeled as statistically independent. Thus the same transmit symbol (say $x[m]$) gets received multiple times, each over a statistically independent channel, (in this case, at times $m, m+1, \ldots, m+L-1$).

A Single Bit Over a Frequency Diversity Channel

Suppose we have just a single bit to transmit. The simplest way to do this is to transmit the appropriate symbol $x = \pm\sqrt{E}$ and stay silent. : i.e., we set

$$x[1] = \pm\sqrt{E}, \tag{2}$$

$$x[m] = 0 \quad m > 1. \tag{3}$$

At the receiver, we have L (complex) voltages that all contain the same transmit symbol immersed in multiplicative and additive noises:

$$y[\ell+1] = h_\ell x + w[\ell+1], \quad \ell = 0, \ldots, L-1. \tag{4}$$

As before, we suppose coherent reception, i.e., the receiver has full knowledge (due to accurate channel tracking) of the channel coefficients $h_0, \ldots, h_{L-1}$. We see that the situation is entirely similar to that of the time diversity channel with repetition coding (as seen in the previous lecture). Then, the appropriate strategy at the receiver (as seen several times, including in the previous lecture on time diversity) is to match filter:

$$y^{\mathrm{MF}} \stackrel{\mathrm{def}}{=} \Re\left[\sum_{\ell=0}^{L+1} h_\ell^* y[\ell+1]\right] \tag{5}$$

$$= \left(\sum_{\ell=0}^{L-1} |h_\ell|^2\right) x + \tilde{w}. \tag{6}$$

Diversity Gain

Here $\tilde{w}$ is real Gaussian with zero mean and variance

$$\left(\sum_{\ell=1}^{L} |h_\ell|^2\right) \frac{\sigma^2}{2}. \tag{7}$$

Thus, the average error probability is generalizes to

$$P_e = \mathbb{E}\left[Q\left(\sqrt{2\mathsf{SNR}\left(\sum_{\ell=0}^{L-1} |h_\ell|^2\right)}\right)\right]. \tag{8}$$

Again, there is an exact expression for the unreliability level when the channel coefficients are independent Rayleigh distributed (just as in the time diversity case):

$$\left(\frac{(1-\mu)}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k, \tag{9}$$

where μ is as in the previous lecture. Finally, we can look for a high SNR approximation; as earlier, we have:

$$P_e \approx \binom{2L-1}{L} \frac{1}{(4A\mathsf{SNR})^L}. \tag{10}$$

The diversity gain is now L: doubling of SNR reduces the unreliability by a factor of

$$\frac{1}{2^L}. \tag{11}$$

OFDM

The approach so far has been very simple: by being silent over $L-1$ successive symbols for every single symbol transmission we have converted the frequency diversity channel into a time diversity one with repetition coding. While this allowed for easy analysis (we could readily borrow from our earlier calculations) and easy harnessing of frequency diversity (so reliability really improved) it has a glaring drawback: we are only transmitting once every L symbols and this is a serious reduction in data rate. If we choose not to stay silent, then we have to deal with ISI while still trying to harness frequency diversity. A natural way to do this is to use OFDM. This way, we convert (at an OFDM block level) to the channel:

$$\tilde{y}_n = \tilde{h}_n \tilde{x}_n + \tilde{w}_n, \quad n = 0, \dots, N_c - 1. \tag{12}$$

This looks very much like the time diversity channel, except that the index n represents the "tones" or "sub-channels" of OFDM.

It is interesting to ask what diversity gain is possible from this channel. Well, since the channel really is the one we started out with (cf. Equation (1)), no more than L-order frequency diversity gain should be possible (there are only L independent channels!). However, a cursory look at Equation (12) suggests that there might be N_c order diversity gain, one for each sub-channel. Clearly this cannot be correct since N_c is typically much larger than L in OFDM.

The catch is that the OFDM channel coefficients $\tilde{h}_0, \dots, \tilde{h}_{N_c-1}$ are *not* statistically independent. They are the discrete Fourier transform output of the input vector

$$[h_0, \dots, h_{L-1}, 0, 0, \dots, 0]\,. \tag{13}$$

Thus there are only about L "truly" independent OFDM channel coefficients and the diversity gain is restricted. Roughly speaking, once in every N_c/L tones fades independently. Since the "bandwidth" of each tone is about $\frac{1}{W}$ (where W is the total bandwidth of communication) we can say that the random discrete time frequency response $H(f)$ is independent over frequencies that are apart by about W/L. This separation of frequency over which the channel response is statistically independent from one another is known as the *coherence bandwidth* (analogous to the coherence time we have seen earlier).

The coherence bandwidth restricts the total amount of frequency diversity available to harness. Physically, this is restricted by the delay spread and communication bandwidth (which decides the sampling rate). Most practical wireless systems harness frequency diversity by a combination of having wide enough bandwidth of communication or a narrowband of communication but one that is hopped over different carrier frequencies.

Looking Ahead

In the next lecture we will see yet another form of diversity: spatial. It is harnessed by having antennas, at both transmitter and receiver.