ECE 361: Fundamentals of Digital Communications

Lecture 24: Antenna Diversity

Introduction

Time diversity comes about from mobility, which may not always be present (at least over the time scale of the application data being communicated). Frequency diversity comes about from good delay spread (relative to the bandwidth of communication) and may not always be available. Fortunately, there is another mode of diversity, *spatial*, that can be harnessed even under stationary scenarios with narrow delay spread. Antennas, transmit and/or receive, are a natural element to capture spatial diversity. Antenna diversity is a "form of time diversity" because instead of a single antenna moving in time we have multiple antennas capturing the same mobility by being in different points in space. This means that antenna *and* time diversity do not really occur independent of each other (while time and frequency diversity occur quite independent of each other).

Antennas come in two forms: receive and transmit. Both forms provide diversity gain, but the nature of harnessing them are quite different. Getting a better understanding of how this is done is the focus of this lecture. We will also get to see the differences between transmit and receive antennas. To be able to focus on the new aspect antennas bring into the picture, our channel models feature *no* time and frequency diversity.

Receive Antenna Channel

The single transmit and multiple receive antenna channel is the following (with no time and frequency diversity):

$$y_\ell[m] = h_\ell x[m] + w_\ell[m], \quad \ell = 1, \dots, L, \quad m \geq 1. \tag{1}$$

Here L is the number of received antennas. Coherent reception is supposed, as usual. We observe that the channel in Equation (1) is identical to the transmit diversity one *with* repetition coding. So, the best strategy for the receiver is to match filter the L voltages:

$$y^{\mathrm{MF}}[m] \stackrel{\mathrm{def}}{=} \Re\left[\sum_{\ell=1}^{L} h_\ell^* y_\ell[m]\right], \tag{2}$$

$$= \left(\sum_{\ell-1}^{L} |h_\ell|^2\right) x[m] + \tilde{w}[m]. \tag{3}$$

As earlier, $\tilde{w}[m]$ is real noise voltage with Gaussian statistics: zero mean and variance equal to

$$\left(\sum_{\ell-1}^{L} |h_\ell|^2\right) \frac{\sigma^2}{2}. \tag{4}$$

Even though the scenario looks eerily like time diversity with repetition coding (indeed, the calculations are rather identical), there are two key differences:

- the "repetition" was done by nature in the spatial dimension. So, no time was wasted in achieving the "repetition". This means that there is no corresponding loss in data rate.

- since there was no actual repetition at the transmitter, there is none of the corresponding loss of transmit power. In other words, we receive more energy than we did in the single antenna setting. Of course, this model makes sense only when we have a small number of antennas and the energy received per antenna is a small fraction of the total transmit energy. Indeed, this model will break down if we have a huge number of antennas: surely, we cannot receive more signal energy than transmitted.

If the antennas are spaced reasonably apart (say about a wavelength distance from each other), then a rich multipath environment will result in statistically independent channel coefficients $h_1, \ldots, h_L$. This means that we have the L fold diversity gain as earlier. It is important to bear in mind that this diversity gain is in *addition* to the L fold *power* gain that was obtained due to the L receive antennas (these resulted in an L fold increase in receive energy, as compared with the single antenna situation).

To summarize:

> receive antennas are just an unalloyed good. They provide both power and diversity gains.

Transmit Antenna Channel

The multiple transmit, single receive antenna channel (with no time and frequency diversity) is modeled as:

$$y[m] = \sum_{\ell=1}^{L} h_\ell x_\ell[m] + w[m], \quad m \geq 1. \tag{5}$$

In other words, L distinct transmit (complex) voltages result in a single receive (complex) voltage. Again, if the antennas are spaced far enough apart (say a distance of one wavelength) in a good multipath environment then an appropriate statistical model would be to consider $h_1, \ldots, h_L$ to be statistically independent (and complex Gaussian distributed).

How does one harness the transmit diversity gain? Naive repetition coding, transmitting the same symbol over each antenna, is *not* going to work. To see this, suppose

$$x_\ell[m] = \frac{x[m]}{\sqrt{L}}, \quad \ell = 1, \ldots, L. \tag{6}$$

Here we have normalized by L to have the same total transmit power as if we had a single transmit antenna. Then the received signal is

$$y[m] = \left(\sum_{\ell=1}^{L} \frac{h_\ell}{\sqrt{L}} \right) x[m] + w[m]. \tag{7}$$

The "effective" channel

$$\left(\sum_{\ell=1}^{L} \frac{h_\ell}{\sqrt{L}} \right) \tag{8}$$

is also Rayleigh fading: a sum of independent complex Gaussian random variables is still a complex Gaussian random variable!

But a small twist will serve to fix the problem: we can repeat over the antennas, but use *only one at a time*. Consider the following strategy:

$$x_\ell[m] = \begin{cases} x & \text{if } \ell = m \\ 0 & \text{else.} \end{cases} \tag{9}$$

Then the first L received symbols are

$$y[\ell] = h_\ell x + w[\ell], \quad \ell = 1, \ldots, L. \tag{10}$$

This scenario is now identical to that of the receive antenna channel (cf. Equation (1)). So, the diversity gain is readily harnessed (by matched filtering at the receiver). But two important differences are worth highlighting:

- there is no power gain from transmit antennas (this is because we actually transmitted L times more energy than in the receive antenna scenario);
- there is a loss in data rate by a factor of $1/L$ (this is because we actually used the transmit one antenna at a time).

Modern communication techniques offer ways of using all the transmit antennas simultaneously while still allowing the harnessing of transmit diversity. This is the area of *space time coding*. While this is a bit beyond the scope of this lecture, we can point out a simple scheme that harnesses transmit diversity without sacrificing the data rate entailed by using one antenna at a time. Consider the following transmission scheme:

$$x_\ell[m] = x_{\ell-1}[m-1], \quad \ell > 1, \quad m > 2. \tag{11}$$

With this transmission technique, the received signal is

$$y[m] = \sum_{\ell=1}^{L} h_\ell x[m - \ell - 1] + w[m]. \tag{12}$$

In other words we have converted the transmit antenna channel into one with frequency diversity. Now a technique such as OFDM will allow the harnessing of the available diversity gain. This technique, known as *delay diversity*, was one of the early space time coding techniques.

Looking Ahead

All wireless systems harness one or more forms of diversity. This is the key to improving the reliability of wireless communication. In the next (and final) lecture, we will see the key new challenges (beyond those that arise in reliable point to point wireless communication) in building a wireless *system* where there are many users communication at the same time. We study these challenges in the context of the cellular wireless system.