ECE398 Spring 2008 Midterm 1

Midterm Instructions:

- All information bits are equally likely to be 0 or 1 and statistically independent of each other.
- No calculators or electronic devices are allowed.
- The test is in class and is closed book and closed notes; a single A4 cheat sheet is allowed.
- Please order your answers in the same order as the questions; this will help us while grading the exam.

Midterm Questions:

1. A transmitted symbol x is equally likely to be $+1$ or -1. It is received at n antennas:

$$y_i = x + z_i, \quad i = 1, \ldots, n. \tag{1}$$

Here the additive noise z_i at antenna i is statistically Gaussian with mean zero and variance σ^2. The additive noises across the antennas are statistically independent of each other and of the transmitted symbol x.

   (a) Find explicitly the structure of the ML receiver that takes the n antenna outputs and makes a decision on whether x is $+1$ or -1. Just state the answer; no need to justify it mathematically. *Hint*: Compare this setting with the repetition coding discussion in Lecture 4.

   (b) Find an expression for the error probability corresponding to the ML receiver. You should use the $Q(\cdot)$ function notation.

   (c) Suppose σ^2 is small. How does the error probability decrease if we double the number of receive antennas?

2. Consider communicating a large information packet reliably and as rate efficiently as possible over a discrete time AWGN channel with SNR (signal to noise ratio) equal to SNR. For each of the following statements, mark whether they are correct or not. Justify your answer briefly.

   (a) At high SNRs, doubling the SNR increases the largest rate of reliable communication by about 0.5 bit per channel use.

   (b) At low SNRs, doubling the SNR increases the largest rate of reliable communication by 1 bit per channel use.

   (c) It is desired that the unreliability of communication of a large packet be no more than p. Then:

i. There is a finite limit called the capacity to the data rate of communication (measured in bits per channel use) when p is set arbitrarily close to zero.

ii. When p is set to a nonzero constant (say 0.25), then it is possible to communicate at data rates larger than capacity.

(d) Suppose we want to consume as low energy as possible for every bit reliably communicated. Then we should use the largest power setting allowed.

3. Consider communicating one bit at a time over an *erasure* channel: the transmitter sends a bit at any time instant and the receiver either gets the bit correctly or knows that the bit got erased. We want to send the four information bits over five time instants. We know, a priori, that exactly one of the received bits will get erased (which one gets erased is known only after the experiment is conducted).

   An engineer decides to use a simple linear code to protect the information bits. Denote the information bits by $b_1, \ldots, b_4$ and the coded bits by $c_1, \ldots, c_5$. Then the engineer is looking for a code matrix $\mathbf{X}$ that maps the information bits to the coded bits in the following linear way:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \\ x_{51} & x_{52} & x_{53} & x_{54} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}. \tag{2}$$

   The entries x_{ij} of the code matrix $\mathbf{X}$ are all either 0 or 1.

   (a) Construct the code $\mathbf{X}$ such that you can reconstruct the original information bits $b_1, \ldots, b_4$ from *any* four of the coded bits $c_1, \ldots, c_5$. *Hint*: Recall the *parity* code we constructed during the erasure channel discussion in Lecture 7.

   (b) Give an explicit method to reconstruct the original information bits $b_1, \ldots, b_4$ from *any* four of the coded bits $c_1, \ldots, c_5$. Of course, this method will depend on how you construct the code $\mathbf{X}$.