7.9

From eq. (7.1.1) $I_c = \dfrac{q D_n n_i^2 A_E e^{qV_{BE}/kT}}{\int_0^{X_B} p\,dx}$

Now $p = N_a + n'(x) \simeq N_a + n(x)$ in high-level injection. Since $n(0)$ dominates the integral over p, we make the approximation

$$\int_0^{X_B} p\,dx = \int_0^{X_B} (N_a + n'(x))\,dx \simeq \int_0^{X_B} (N_a + n(x))\,dx,$$

where $n(x) \approx n(0)\left[1 - \frac{x}{X_B}\right]$ {linear}

Then: $\int_0^{X_B} N_a\,dx = N_a X_B + \frac{1}{2} n(0)[X_B] = N_a X_B \left[1 + \frac{n(0)}{2N_a}\right]$

Defining $I_F = \dfrac{q \tilde{D}_n n_i^2 A_E e^{qV_{BE}/kT}}{N_a X_B}$ then $I_c = \dfrac{I_F}{1 + \frac{n(0)}{2N_a}}$

Eq. (7.2.3) for $n(0)$ can be written

$n(0) = \frac{N_a}{2}\left[\left(1 + \frac{4I_F}{I_k}\right)^{1/2} - 1\right]$ where $I_k = \dfrac{q\tilde{D}_n A_E N_a}{X_B}$

$\therefore I_c = \dfrac{I_F}{\left[1 + \frac{1}{4}\left(1 + \frac{4I_F}{I_k}\right)^{1/2} - 1\right]}$ Since I_B is a result of injected holes into the emitter.

$\therefore I_B = \dfrac{I_F}{\beta_{F0}}$ $\therefore \beta_F = \dfrac{I_c}{I_B} = \dfrac{\beta_0}{\left[1 + \frac{1}{4}\left[\left(1 + \frac{4I_F}{I_k}\right)^{1/2} - 1\right]\right]}$ $\left[\dfrac{\beta_F}{\beta_0} = \dfrac{I_c}{I_F}\right]$

For the constants given: $I_k = 16\,mA$

$I_F \ll I_k \Rightarrow \beta_F \to \beta_0$; $I_F \gg I_k \Rightarrow \beta_F \to 2\beta_0 \sqrt{\frac{I_k}{I_F}}$

At $I_F = I_k$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{\left\{1 + \frac{1}{4}\left[(1+4)^{1/2} - 1\right]\right\}} = \frac{1}{1.31} = 0.76$$

and $\dfrac{I_c}{I_F} = \dfrac{\beta_F}{\beta_0} \Rightarrow I_c = 0.76\, I_k$

At $I_F = 2I_k$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{(1 + \frac{1}{4}(2))} = \frac{1}{1.5} = 0.67$$

$I_c = 2 \times I_k \times 0.67 = 1.34\, I_k$

At $I_F = 10 I_k$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{(1 + \frac{1}{4}[\sqrt{41} - 1])} = 0.425$$

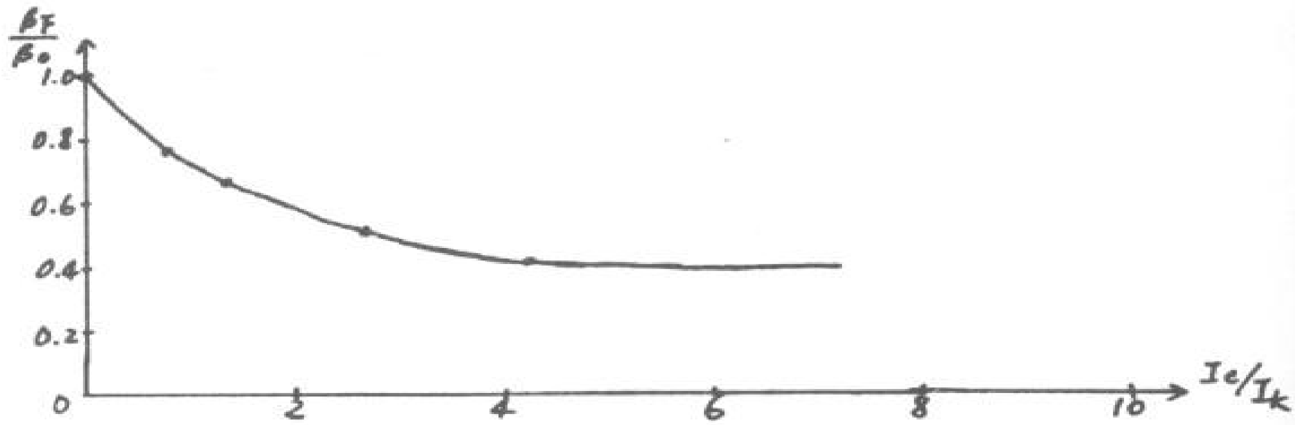
$I_c = 10 \times I_k \times 0.425 = 4.25\, I_k$

At $I_F = 5\, I_k$,

At $I_F = 5 I_k$,

$$\frac{\beta_F}{\beta_0} = \frac{1}{\left(1 + \frac{1}{4}(\sqrt{21} - 1)\right)} = 0.527$$

$$I_c = 5 \times I_k \times 0.527 = 2.63 I_k$$



7.11

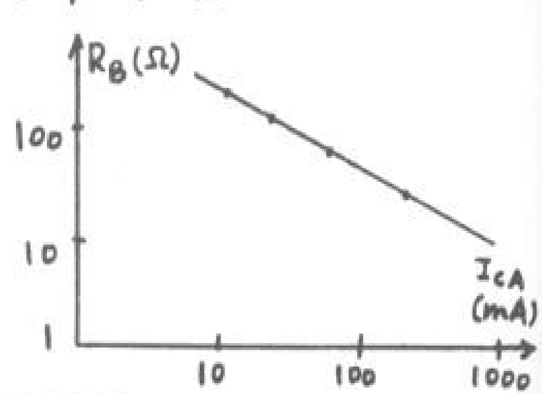
(a) $I_{cA} = I_s \exp\left(\frac{V_{BE} - \frac{I_{cA}}{\beta_F} R_B}{V_t}\right)$ Eq. (7.2.13) - - - - - - - - - (1)

$I_{cI} = I_s \exp\left(\frac{V_{BE}}{V_t}\right)$ - - - - - (2)

(2)/(1) $\frac{I_{cI}}{I_{cA}} = \exp\left(\frac{I_{cA} R_B}{\beta_F V_t}\right)$ or $R_B = \frac{V_t \beta_F}{I_{cA}} \ln\left(\frac{I_{cI}}{I_{cA}}\right)$

(b) With $I_S = 3 \times 10^{-14}$ A, $V_t = 0.0252$ Volts, $\beta_F = 100.0$

V_{BE} (V)	I_{cA} (mA)	I_{cI} (mA)	R_B (Ω)
0.70	11.25	35	253
0.72	22.4	77	139
0.75	56.2	253	67
0.80	200	1838	28



7.14

(a) From eq. (7.3.8) $\tau_B = \frac{x_B^2}{\tilde{D}_n}\left[\int_0^1 \frac{1}{P(y)}\left(\int_y^1 P(\xi)\,d\xi\right)dy\right]$

For exponential doping and low-level injection

$P(x) = N_a(x) = N_{a0}e^{-x/\ell}$. If we let $y = \frac{x}{x_B}$ and define $K = \frac{x_B}{\ell}$

then $P(y) = N_{a0}e^{-Ky}$

$\therefore \int_y^1 P(\xi)\,d\xi = N_{a0}\int_y^1 e^{-K\xi}d\xi = \frac{N_{a0}}{K}\left[e^{-Ky} - e^{-K}\right]$ and

$\int_0^1 \frac{1}{P(y)}\left(\int_y^1 P(\xi)\,d\xi\right)dy = \frac{1}{K}\int_0^1 \frac{e^{-Ky} - e^{-K}}{e^{-Ky}}\,dy = \frac{1}{K}\int_0^1\left[1 - e^{K(y-1)}\right]dy$

$= \frac{1}{K}\left[1 - \frac{1}{K} + \frac{e^{-K}}{K}\right] = \frac{1}{K^2}\left[K - 1 + e^{-K}\right]$

$\therefore \tau_B = \frac{x_B^2}{\tilde{D}_n K^2}\left[K - 1 + e^{-K}\right]$

If we write

$$\tau_B = \frac{x_B^2}{\nu \tilde{D}_n} \quad \text{then } \nu = \frac{K^2}{[K - 1 + e^{-K}]}$$

$K \to 0 \Rightarrow \ell \to \infty$ so that the base doping becomes uniform.

$e^{-K} \simeq 1 - K + \frac{K^2}{2} \Rightarrow K - 1 + e^{-K} \simeq \frac{K^2}{2}$ and $\nu = 2$

$\therefore \tau_B = x_B^2/2\tilde{D}_n$ as expected

(b) With $K = 20$ $\nu = \frac{400}{19 + e^{-20}} = 21.05$

We have $\frac{N_a(0)}{N_a(x_B)} = e^K$, since $\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i}$ $\therefore$

$\phi_F(0) - \phi_F(x_B) = \frac{kT}{q}\ln\frac{N_a(0)}{N_a(x_B)} = \frac{kT}{q}K = 0.516\,\text{eV}$ if $K = 20$

Since this value is close to $E_g/2$, $K = 20$ is about as large as can be realized practically.