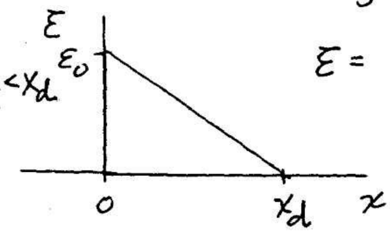
APPENDIX PROBLEMS

A1.1 FOR $0<x<x_d$ $\quad \rho = -\rho_1 \quad \dfrac{d\mathcal{E}}{dx} = -\dfrac{\rho_1}{\epsilon_s}$

(a) $\therefore \mathcal{E} = \mathcal{E}_0 - \dfrac{\rho_1 x}{\epsilon_s}$ $\quad 0<x<x_d$

where $\mathcal{E}_0 = \dfrac{\rho_1 x_d}{\epsilon_s}$

$\mathcal{E} = 0, \; x<0$
$\quad\;\; 0, \; x > x_d$

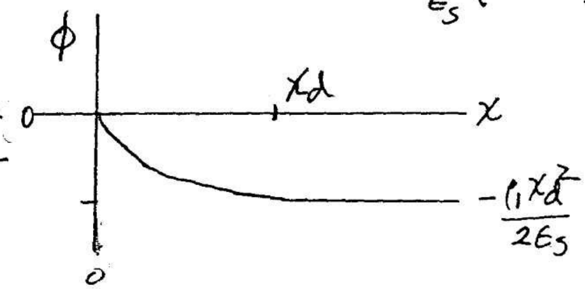


(b) $\phi = -\int \mathcal{E}\, dx$

$\phi = -\mathcal{E}_0 x + \dfrac{\rho_1 x^2}{2\epsilon_s} \quad 0<x<x_d$ or $\phi = -\dfrac{\rho_1}{\epsilon_s}\left(x_d x - \dfrac{x^2}{2}\right)$

Take $\phi(x<0) = 0$

$\phi(x_d) = -\dfrac{\rho_1 x_d^2}{2\epsilon_s}$



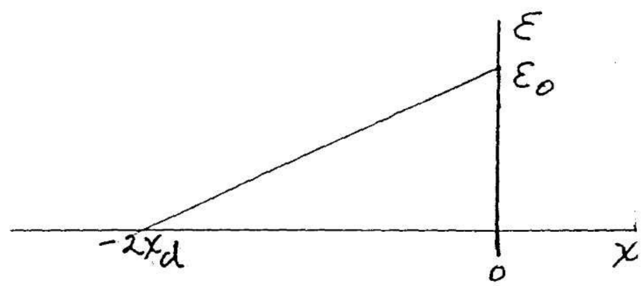
(c) $\phi(0) - \phi_{x_d} = \dfrac{\rho_1 x_d^2}{2\epsilon_s}$

(d) $\mathcal{E} = 0 \begin{cases} x < -2x_d \\ x > 0 \end{cases}$

$\mathcal{E}(x) = \dfrac{\rho_1}{\epsilon_s}\left(x_d + \dfrac{x}{2}\right)$

$-2x_d < x < 0$

$\mathcal{E}(0) = \dfrac{\rho_1 x_d}{\epsilon_s}$

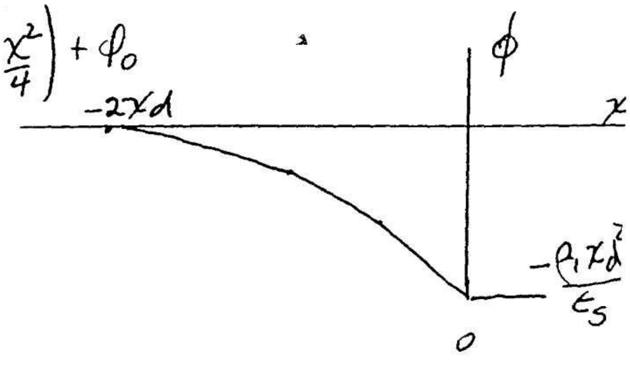


$\phi = -\int \mathcal{E}\, dx = -\dfrac{\rho_1}{\epsilon_s}\left(x_d x + \dfrac{x^2}{4}\right) + \phi_0$

If $\phi_0 = 0 \; (x \le -2x_d)$

$\phi = -\dfrac{\rho_1}{\epsilon_s}\left(x_d + \dfrac{x}{2}\right)^2$

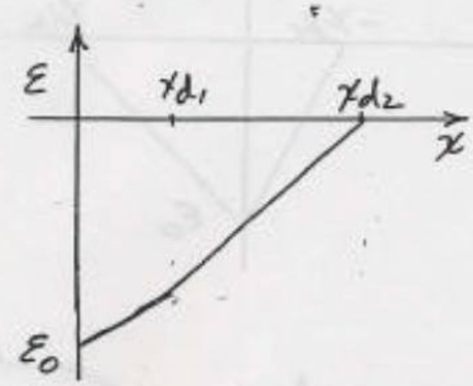
$\therefore \; \phi(-2x_d) - \phi(x=0) = \dfrac{\rho_1 x_d^2}{\epsilon_s}$



A1.2

(a) For $x<0$, $\mathcal{E}=0$.

For $0<x<x_{d1}$ $\quad \frac{d\mathcal{E}}{dx}=\frac{\rho_1}{\epsilon_s}$ $\quad \therefore \mathcal{E}=\mathcal{E}_0+\frac{\rho_1 x}{\epsilon_s}$



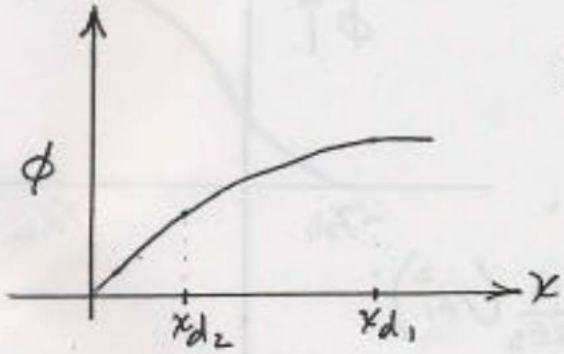
By Gauss' Law $\mathcal{E}_0=-\frac{1}{\epsilon_s}\left[\rho_1 x_{d1}+2\rho_1(x_{d2}-x_{d1})\right]$

$$\mathcal{E}_0=-\frac{\rho_1}{\epsilon_s}\left[2x_{d2}-x_{d1}\right]$$

and $\mathcal{E}(x_{d1})=\mathcal{E}_0 x+\frac{\rho_1 x_{d1}}{\epsilon_s}$

$$\therefore \mathcal{E}(x_{d1})=-\frac{\rho_1}{\epsilon_s}\left[2x_{d2}-x_{d1}-x_{d1}\right]$$

$$=-\frac{2\rho_1}{\epsilon_s}\left[x_{d2}-x_{d1}\right]$$



For $x_{d1}<x<x_{d2}$

$$\mathcal{E}=\mathcal{E}(x_{d1})+\frac{2\rho_1}{\epsilon_s}(x-x_{d1})$$

$$=-\frac{2\rho_1}{\epsilon_s}\left[x_{d2}-x_{d1}-(x-x_{d1})\right]$$

$$=-\frac{2\rho_1}{\epsilon_s}\left[x_{d2}-x\right]$$

(b) $\phi=0 \quad x<0$

$\phi=-\int\mathcal{E}\,dx$ For $0<x<x_{d1}$

$$\phi=\mathcal{E}_0 x-\frac{\rho_1 x^2}{2\epsilon_s}$$

(c) $\phi(0)-\phi(x_{d1})$

$=-\frac{\rho_1}{\epsilon_1}\left(2x_{d2}x_{d1}-\frac{3}{2}x_{d1}^2\right)$

$$=\frac{\rho_1}{\epsilon_s}\left[2x_{d2}x-x_{d1}x-\frac{x^2}{2}\right]$$

$$\phi(x=x_{d1})=\frac{\rho_1}{\epsilon_s}\left[2x_{d2}x_{d1}-\frac{3}{2}x_{d1}^2\right]$$

$\phi(0)-\phi(x_{d2})$

$=-\frac{\rho}{\epsilon_s}\left(x_{d2}^2-\frac{x_{d1}^2}{2}\right)$

For $x_{d1}<x<x_{d2}$

$$\phi=\phi(x_{d1})-\int_{x_{d1}}^{x}\mathcal{E}\,dx$$

$$=\frac{\rho}{\epsilon_s}\left(2x_{d2}x-\frac{x_{d1}^2}{2}-x^2\right)$$

$$\phi(x_{d2})=\frac{\rho}{\epsilon_s}\left(x_{d2}^2-\frac{x_{d1}^2}{2}\right)$$