4.2

One-sided step junction : $\mathcal{E}_{max} = \frac{q}{\epsilon_s} N x_d$, $x_d = \frac{\mathcal{E}_{max}\,\epsilon_s}{q N_a}$

$\mathcal{E}_{max} = \frac{2(\phi_i - V_a)}{x_d}$, $\phi_i - V_a = \frac{x_d\,\mathcal{E}_{max}}{2}$, $V_a = \phi_i - \frac{x_d\,\mathcal{E}_{max}}{2}$

where $\phi_i = \frac{1}{2}E_g + kT \ln \frac{N}{n_i}$

	$N(cm^{-3})$	$\mathcal{E}_{max}$ (V/cm) (From Fig 4.12)	$X_d(\mu m)$	ϕ_i (V)	V_a (V)
(a)	10^{15}	3.1×10^{5}	20.1	0.85	-311
(b)	10^{16}	4.2×10^{5}	2.7	0.91	-55.8
(c)	10^{17}	6.2×10^{5}	0.40	0.97	-11.4
(d)	10^{18}	1.3×10^{6}	0.084	1.03	-4.43

4.5

(a) From Poisson's Eq. $\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$ and $\phi = -\int \mathcal{E}\,dx$

Region	$x_1 < x < x_2$	$x_2 < x < x_3$	$x_3 < x < x_4$
$\rho(x)$	qN_d	0	$-qN_a$
$\mathcal{E}(x)$	$\frac{qN_d}{\epsilon}(x-x_1)$	$\mathcal{E}_{max} = \frac{qN_d}{\epsilon}(x_2-x_1) = \frac{qN_a}{\epsilon}(x_4-x_3)$	$\frac{qN_a}{\epsilon}(x_4-x)$
$\phi(x)$	$-\frac{qN_d}{2\epsilon}(x-x_1)^2$	$-\phi_n - \frac{qN_d}{\epsilon}(x_2-x_1)(x-x_2)$	$-\phi_n - \phi_o - \phi_p + \frac{qN_a}{2\epsilon}(x_4-x)^2$

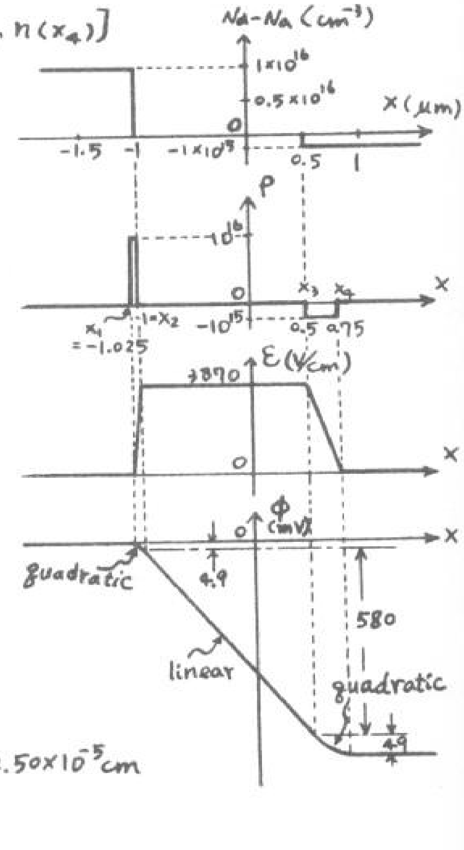
where $\phi_n = \frac{qN_d}{2\epsilon}(x_2-x_1)^2$, $\phi_o = \frac{qN_d}{\epsilon}(x_2-x_1)(x_3-x_2)$, and $\phi_p = \frac{qN_a}{2\epsilon}(x_4-x_3)^2$

Need to find the depletion region widths : $x_2 - x_1$ and $x_4 - x_3$

$$\phi_i = \phi_n + \phi_o + \phi_p = \frac{kT}{q}\left[\ln n(x_1) - \ln n(x_4)\right]$$
$$= \frac{kT}{q}\left[\ln N_d - \ln \frac{n_i^2}{N_a}\right]$$
$$= \frac{kT}{q}\left[\ln N_d + \ln N_a - \ln n_i^2\right]$$
$$= \frac{kT}{q}\ln\frac{N_d N_a}{n_i^2}$$ just as for pn junction
$$= 0.637\,V$$
$$= \frac{qN_d}{2\epsilon}(x_2-x_1)^2 + \frac{qN_d}{\epsilon}(x_2-x_1)(x_3-x_2) + \frac{qN_a}{2\epsilon}(x_4-x_3)^2$$

By charge neutrality :

$$qN_d(x_2-x_1) = qN_a(x_4-x_3)$$
$$\phi_i = \frac{qN_d}{2\epsilon}(x_2-x_1)^2 + \frac{q N_d^2}{2\epsilon N_a}(x_2-x_1)^2 + \frac{qN_d}{\epsilon}(x_2-x_1)(x_3-x_2)$$
$$[7.73\times10^{8} + 7.73\times10^{9}](x_2-x_1)^2 + 2.32\times10^{5}(x_2-x_1) - 0.637 = 0$$
$$\therefore (x_2-x_1)^2 + 2.73\times10^{-5}(x_2-x_1) - 7.50\times10^{-11} = 0$$
$$\therefore x_2 - x_1 = 2.50\times10^{-6}\,cm,\quad x_4 - x_3 = 2.50\times10^{-5}\,cm$$



(b) $\mathcal{E}_{max} = \frac{qN_d}{\epsilon}(x_2 - x_1) = 3870$ V/cm

For pn junction, Eq (4.3.3) $\mathcal{E}_{max} = \frac{2\phi_i}{x_d}$

Eq. (4.3.1) $x_d = \left[\frac{2\epsilon_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\phi_i\right]^{1/2} = 9.52 \times 10^{-5}$ cm

and $\mathcal{E}_{max} = 13400$ V/cm (3.5 times $\mathcal{E}_{max}$ for pin junction)

(c) Much of the built-in voltage is dropped across the intrinsic region, so that the depletion regions in the n- and p-type material need not extend as far as in a p-n junction. The maximum field, which increases linearly with the extent of the space charge region in the doped material is, therefore, reduced. Alternatively, we may say that the total depletion-region width is the intrinsic region plus some depleted regions in the n- and p-type material. This longer depletion region decreases the maximum field.

(d) $C = \left|\frac{dQ}{dV_a}\right| = qN_d\left|\frac{d(x_2 - x_1)}{dV_a}\right|$. From part (a)

$$a(x_2 - x_1)^2 + b(x_2 - x_1) - (\phi_i - V_a) = 0$$

$$x_2 - x_1 = \frac{-b + \sqrt{b^2 + 4a(\phi_i - V_a)}}{2a}, \quad \frac{d(x_2 - x_1)}{dV_a} = \frac{1}{4a}\frac{-4a}{\sqrt{b^2 + 4a(\phi_i - V_a)}}$$

where $a = \frac{qN_d}{2\epsilon}\left(1 + \frac{N_d}{N_a}\right)$, $b = \frac{qN_d}{\epsilon}(x_3 - x_2)$

$$\therefore C = \frac{1}{\left[\left(\frac{x_3 - x_2}{\epsilon}\right)^2 + \frac{2}{q\epsilon}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)(\phi_i - V_a)\right]^{1/2}}$$

The capacitance has the same functional dependence as for a p-n junction except for the added term under the square root because of the wider depletion region. This added term corresponds to the intrinsic region and, therefore, does not vary with voltage. The other term corresponds to depletion in the doped regions and, consequently, is voltage dependent. The capacitance of the pin structure is much lower because the depletion region is wider.

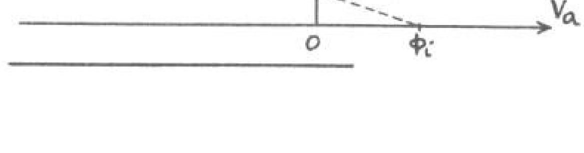
For a p-n junction the above expression reduces to

$$C = \left[\frac{q\epsilon}{2\left(\frac{1}{N_d} + \frac{1}{N_a}\right)(\phi_i - V_a)}\right]^{1/2}$$ which is Eq. (4.3.8)

For pin, $\frac{1}{C^2} = \frac{(x_3 - x_2)^2}{\epsilon^2} + \frac{2}{q\epsilon}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)(\phi_i - V_a)$

$= 2.10 \times 10^{16} + 1.33 \times 10^{16}(\phi_i - V_a)$

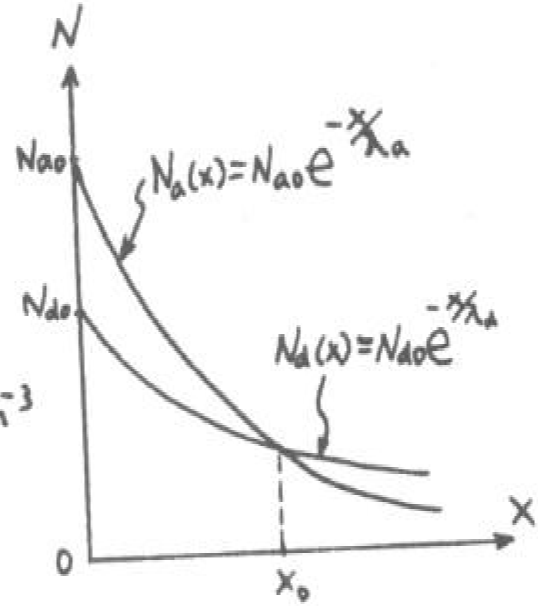
For pn, $\frac{1}{C^2} = 1.33 \times 10^{16}(\phi_i - V_a)$



4.8

$x_0 = 10^{-4}$ cm, $\lambda_a = 10^{-4}$ cm, $\lambda_d = 2\times10^{-4}$ cm

$N_{a0} = 10^{18}$ cm^{-3}



(a) at junction $x = x_0$

$$N_{d0} e^{-\frac{x_0}{\lambda_d}} = N_{a0} e^{-\frac{x_0}{\lambda_a}}$$

$$N_{d0} = N_{a0} e^{x_0\left(\frac{1}{\lambda_d} - \frac{1}{\lambda_a}\right)} \approx 6.1\times10^{17}\ \text{cm}^{-3}$$

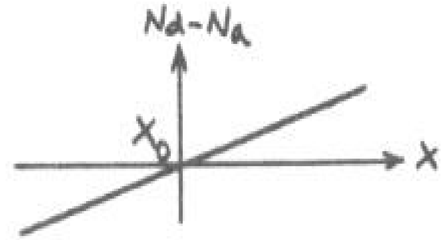
(b) $N_d - N_a \simeq a(x - x_0)$

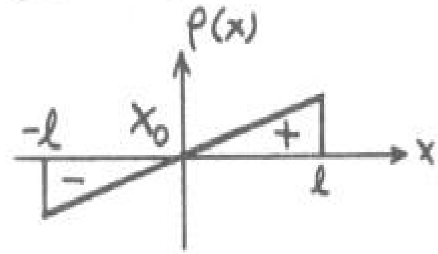
where $a = \frac{d}{dx}(N_d - N_a)\big|_{x=x_0}$

$$= -\frac{N_{d0}}{\lambda_d} e^{-\frac{x_0}{\lambda_d}} + \frac{N_{a0}}{\lambda_a} e^{-\frac{x_0}{\lambda_a}} \approx 1.83\times10^{21}\ \text{cm}^{-4}$$

Space charge density

$\rho(x) = q(N_d - N_a) \approx qa(x - x_0)$





(c) $x_d = \left(\frac{12\epsilon_s}{qa}\right)^{\frac{1}{3}} (\phi_i - V_a)^{\frac{1}{3}}$ Eq. (4.3.2)

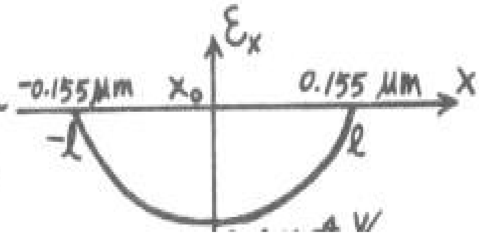
$\mathcal{E}_{max} = \frac{3(\phi_i - V_a)}{2x_d}$ Eq. (4.3.4)

At thermal equilibrium $V_a = 0$,

for $\phi_i = 0.7$ V,

$$x_d = \left(\frac{12\times11.7\times8.85\times10^{-14}}{1.6\times10^{-19}\times1.84\times10^{21}}\right)^{\frac{1}{3}} (0.7)^{\frac{1}{3}} \simeq 3.1\times10^{-5}\ \text{cm}.$$

$$\mathcal{E}_{max} = \frac{3\times0.7}{2\times3.1\times10^{-5}} \simeq 3.4\times10^{4}\ \text{V/cm}$$



4.9