6.3

Let $n_e(x)$ denote the electron density in the case of exponential base doping and $n_c(x)$ denote the density in the case of constant doping. Using the result of Prob. 6.2 with $N_a(x) = N_a(0)e^{-x/\mathcal{L}}$ gives

$$I_c = \frac{qA_E\tilde{D}_n(x) \cdot N_a(0) e^{-x/\mathcal{L}} n_e(x)}{N_a(0)\int_x^{x_B} e^{-x/\mathcal{L}}\,dx'} = \frac{qA_E\tilde{D}(x)e^{-x/\mathcal{L}} n_e(x)}{\mathcal{L}[e^{-x/\mathcal{L}} - e^{-x_B/\mathcal{L}}]}$$

Solving for $n_e(x)$, gives:

$$n_e(x) = \frac{I_c\mathcal{L}[1-\exp(\frac{x-x_B}{\mathcal{L}})]}{qA_E\tilde{D}_n(x)}, \text{ using } n_e(0) = \frac{I_c\mathcal{L}[1-\exp(\frac{-x_B}{\mathcal{L}})]}{qA_E\hat{D}_n(0)}$$

then $$n_e(x) = n_e(0)\frac{\tilde{D}_n(0)}{\tilde{D}_n(x)}\left[\frac{1-\exp(\frac{x-x_B}{\mathcal{L}})}{1-\exp(\frac{-x_B}{\mathcal{L}})}\right]$$

As $\mathcal{L}\to\infty$ the base doping becomes constant. Hence as $\mathcal{L}\to\infty$ $n_e(x)\to n_c(x)$, $\tilde{D}_n(x)\to D_n$, and $\exp(\frac{x-x_B}{\mathcal{L}}) \to 1+\frac{x-x_B}{\mathcal{L}}$

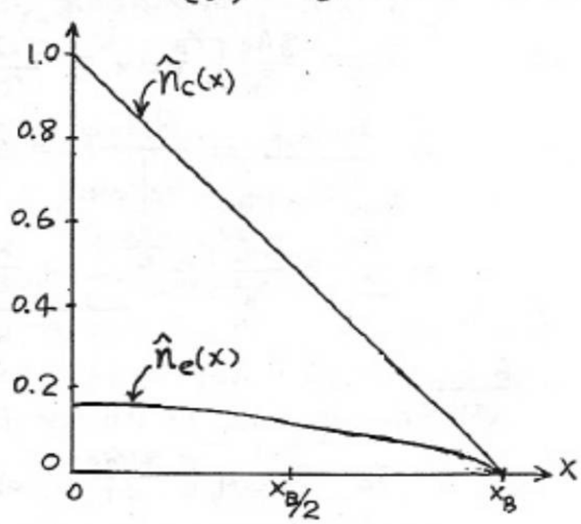
$\therefore$ $n_c(x) = \frac{n_c(0)}{x_B}(x_B - x)$, where $n_c(0) = \frac{I_c x_B}{qA_E D_n}$

Since the exponential doping variation of prob. 6.1 is not very large $\hat{D}_n(x) \simeq D_n$. If we normalize both densities with respect to $n_c(0)$; i.e., if we let $\hat{n}(x) = n(x)/n_c(0)$ we obtain

$$\hat{n}_e(x) = \hat{n}_e(0)\left[\frac{1-\exp(\frac{x-x_B}{\mathcal{L}})}{1-\exp(\frac{-x_B}{\mathcal{L}})}\right] = \frac{\mathcal{L}}{x_B}\left[1-\exp(\frac{x-x_B}{\mathcal{L}})\right]$$

and $\hat{n}_c(x) = 1 - \frac{x}{x_B}$ where $\hat{n}_e(0) = \frac{n_e(0)}{n_c(0)} = \frac{\mathcal{L}}{x_B}[1-\exp(\frac{-x_B}{\mathcal{L}})]$

$\hat{n}_c$	$\hat{n}_e$	x
1	0.137	0
0.5	0.054	$x_B/2$
0	0	x_B



$J_{ndrift}(x_B) \propto n(x_B)$ and $n(x_B) \simeq 0$

Thus the total current at $x = x_B$ is a diffusion current no matter how the base is doped. With all other parameters equal, the only way to have the same current in the two cases is to have the same gradient at $x = x_B$, since $J_{ndiff.} \propto dn/dx$

6.4

The total stored minority charge is given by $Q_B = -qA_E\int_0^{x_B} n(x)\,dx$

For the exponential doping we have from prob. 6.3

$$n_e(x) = \frac{n_e(0)}{\left[1-e^{\frac{-x_B}{\mathcal{L}}}\right]}\left[1-e^{\frac{(x-x_B)}{\mathcal{L}}}\right]$$

$$\therefore\ Q_B\Big|_{exp.} = \frac{-qA_E n_e(0)}{\left[1-\exp\left(\frac{-x_B}{\mathcal{L}}\right)\right]}\int_0^{x_B}\left[1-\exp\left(\frac{x-x_B}{\mathcal{L}}\right)\right]dx = \frac{-qA_E n_e(0)\mathcal{L}}{\left[1-\exp\left(\frac{-x_B}{\mathcal{L}}\right)\right]}\left[e^{\frac{-x_B}{\mathcal{L}}}-1+\frac{x_B}{\mathcal{L}}\right]$$

Letting $\mathcal{L}\to\infty$ and expanding the exponential terms gives

$$Q_B\Big|_{constant} = \frac{-qA_E n_c(0)x_B}{2}$$

Assume that base current is due only to base recombination.
If I_c is the same in the two transistors

$$\beta = \frac{I_c}{I_B} \Longrightarrow \frac{\beta_c}{\beta_e} = \frac{I_{rB}|_{exp.}}{I_{rB}|_{const.}}$$

Since the injected electron density greatly exceeds the equilibrium electron density over most of the base and lifetime is not strongly x dependent, eq. (6.2.4) can be writtern :

$$I_{rB} = \frac{qA_E}{\tau_n}\int_0^{x_B} n\,dx = \frac{-Q_B}{\tau_n}$$

$$\therefore\ \frac{I_{rB}|_{exp.}}{I_{rB}|_{const.}} = \frac{Q_B|_{exp.}}{Q_B|_{const.}} = \frac{2n_e(0)\mathcal{L}}{n_c(0)x_B}\frac{\left[e^{\frac{-x_B}{\mathcal{L}}}-1+\frac{x_B}{\mathcal{L}}\right]}{\left[1-e^{\frac{-x_B}{\mathcal{L}}}\right]} = \frac{2\mathcal{L}^2}{x_B^2}\left[e^{\frac{-x_B}{\mathcal{L}}}-1+\frac{x_B}{\mathcal{L}}\right]$$

$$\therefore\ \frac{\beta_c}{\beta_e} = \frac{2\mathcal{L}^2}{x_B^2}\left[e^{\frac{-x_B}{\mathcal{L}}}-1+\frac{x_B}{\mathcal{L}}\right] = 0.340$$

6.20

$dt = \frac{dx}{v(x)}$ so that $\tau_B = \int_0^{x_B}\frac{dx}{v(x)}$

Since $I_c = qn(x)v(x)A$, $v(x) = \frac{I_c}{qn(x)A}$ $\therefore\ \tau_B = \frac{qA}{I_c}\int_0^{x_B} n(x)\,dx$

For the prototype transistor, we have from Prob. 6.3

$$n_c(x) = \frac{n_c(0)}{x_B}(x_B-x) = \frac{I_c}{qA_E D_n}(x_B-x)$$

$$\therefore\ \tau_B = \frac{qA_E\ I_c}{I_c qA_E D_n}\int_0^{x_B}(x_B-x)\,dx = \frac{1}{D_n}\left[x_B^2-\frac{x_B^2}{2}\right]$$

$$\therefore\ \tau_B = \frac{x_B^2}{2D_n}$$