In the circuit shown in Figure 3.1, it is assumed that the value of the feedback resistor, R_f, is large enough so that it can also serve as one of the two resistors in the voltage divider that develops the base voltage. If RF circuit design considerations dictate that R_f should be chosen to be smaller than is reasonable for the bias network, an additional resistance can be used in series with R_f and this resistor would then be bypassed with a capacitor.

3.2.2 Analysis of input/output impedance and power gain

For approximate analysis of this circuit using pencil and paper, we can use a simplified (low-frequency) hybrid-π model for the transistor and assume that the transformer is an ideal 1:1 transformer, i.e. that the two windings are perfectly coupled. An ideal transformer can be described by two equations. Denoting the voltage across each of the windings by V_1 and V_2 , and the current directed into the dot associated with each of the windings by I_1 and I_2, then an ideal 1:1 transformer satisfies:

$$V_1 = V_2 \tag{3.1}$$

$$I_1 = -I_2 \tag{3.2}$$

The small signal equivalent circuit for the amplifier is shown in Figure 3.2. The ideal transformer relationships have been incorporated into the figure by explicitly showing that the currents flowing through each of the windings of the transformer are equal to I and the voltage across each winding is equal to V_o.

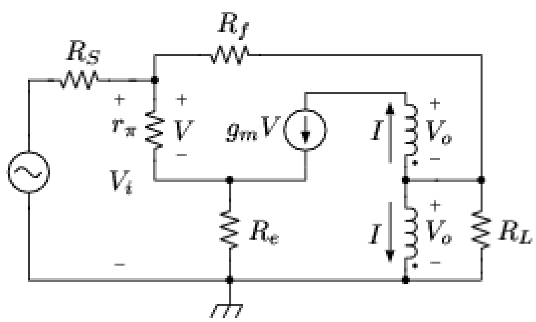

Figure 3.2: Small signal model for the amplifier shown in Figure 3.1.

The input impedance of the amplifier (i.e., the impedance seen by the source with Thevenin impedance R_S) can be shown to be:

$$Z_{in} = \frac{(R_L + R_f)(R_e + r_\pi + g_m R_e r_\pi)}{R_e + r_\pi + R_L + R_f + g_m r_\pi (R_e + 2R_L)} \tag{3.3}$$

The output impedance of the amplifier is:

$$Z_{out} = \frac{(R_S + R_f)(R_e + r_\pi + g_m R_e r_\pi) + R_S R_f}{R_S + R_e + r_\pi + g_m r_\pi (R_e + 2R_S)} \tag{3.4}$$

The voltage gain is (note that V_i is the voltage across the amplifier input terminals, i.e the voltage between the base of the transistor and ground):

$$A_v = \frac{V_o}{V_i} = -2g_m \frac{R_L R_f}{R_L + R_f} \frac{1}{1 + R_e(g_m + \frac{1}{r_\pi})} + \frac{R_L}{R_L + R_f} \tag{3.5}$$

We can now use equations 3.3 through 3.5 to derive values for the independent parameters g_m, R_e, R_f, that will yield useful impedance and gain properties. One of our goals is to design an amplifier that will be simultaneously matched to 50Ω at both ports. To develop a design equation that will enforce the simultaneous conjugate match requirement we specify:

$$Z_{in}|_{R_L=R} = R \tag{3.6}$$

$$Z_{out}|_{R_S=R} = R \tag{3.7}$$

Later, we will set $R = 50\ \Omega$. Using constraints 3.6 and 3.7 in equations 3.3 and 3.4 we obtain two equations:

$$R = \frac{(R+R_f)(R_e + r_\pi + g_m R_e r_\pi)}{R_e + r_\pi + R + R_f + g_m r_\pi (R_e + 2R)} \tag{3.8}$$

$$R = \frac{(R+R_f)(R_e + r_\pi + g_m R_e r_\pi) + RR_f}{R + R_e + r_\pi + g_m r_\pi (R_e + 2R)} \tag{3.9}$$

Multiply both sides of equation 3.8 by the denominator of the right-hand side to obtain:

$$RR_e + Rr_\pi + R^2 + RR_f + g_m r_\pi R(R_e + 2R) = RR_e + Rr_\pi + g_m RR_e r_\pi + R_f R_e + R_f r_\pi + g_m R_e R_f r_\pi$$

Combine terms to obtain:

$$R^2(1 + 2g_m r_\pi) - R_f(R_e + r_\pi + g_m r_\pi R_e - R) = 0 \tag{3.10}$$

Carrying out the same procedure on equation 3.9 yields:

$$R^2(1 + 2g_m r_\pi) - R_f(R_e + r_\pi + g_m r_\pi R_e + R) = 0 \tag{3.11}$$

Equations 3.10 and 3.11 differ in the second term, and they cannot be satisfied simultaneously unless $R \ll R_e + r_\pi + g_m r_\pi R_e$, in which case the equations become essentially identical. In practical applications, the approximation $\beta = g_m r_\pi \gg 1$ will hold, and if we choose values of R_e, R_f, and
Design eqn 1 r_π, such that $R \ll R_e + r_\pi + g_m r_\pi R_e$ equations 3.10 and 3.11 yield, approximately:

$$R_f \simeq \frac{2\beta R^2}{r_\pi + \beta R_e} \tag{3.12}$$

We can view equation 3.12 as one of the design equations for the amplifier, as it specifies what value of feedback resistor must be used once the other parameters have been determined.

Next, consider the voltage gain with $R_L = R$, under the assumption that $\beta \gg 1$:

$$A_v \simeq \frac{-2\beta RR_f}{(R+R_f)(r_\pi + \beta R_e)} + \frac{R}{R+R_f} \tag{3.13}$$

The second term will be less than 1. We will be interested in the parameter regime where the amplifier has voltage gain significantly greater than 1 so this term can be neglected. Since we have specified a simultaneous conjugate match with equal source and load impedances, the power gain is simply the square of the voltage gain. (If we had specified a simultaneous conjugate match with different source and load impedances, this wouldn't be true!) Also, recall that under conjugately matched conditions, the operating, transducer, and available power gains are equal. The power gain of the amplifier is:

$$G = [\frac{-2\beta R R_f}{(R+R_f)(r_\pi + \beta R_e)}]^2 \tag{3.14}$$

So far, the approximations that we have employed are $\beta \gg 1$ and $R \ll R_e + r_\pi + g_m r_\pi R_e$. If we make one more approximation by constraining the value of R_f to be much larger than $R = 50\Omega$, ($R_f \gg R$), then equation 3.14 simplifies to **Design eqn 2**

$$G = (\frac{-2\beta R}{r_\pi + \beta R_e})^2 \tag{3.15}$$

This extra constraint is useful because it makes the gain independent of the feedback resistance R_f. We can use equation 3.15 to choose R_e to set the gain of the amplifier once R, β, and r_π are determined. Then the value required for R_f can be determined using equation 3.12 in order to satisfy the conjugate match criterion. In practice, the value of β will be determined by the transistor that is chosen for the amplifier, and then the quiescent collector current will determine r_π. We are free to choose the value of collector current (and therefore r_π) to satisfy some other criterion such as minimum noise figure, or we may choose to use the collector current that optimizes the transistor's gain-bandwidth product in an attempt to achieve highest gain.

Now, consider some typical numbers. Suppose that $R = 50\,\Omega$, $\beta = 100$, and $I_{CQ} = 10\,\text{mA}$ so that $r_\pi \simeq 250\,\Omega$. To satisfy the approximation $R_f \gg R$, it is reasonable to require that R_f be at least 10 times as large as R, i.e. $R_f \geq 500\,\Omega$. Likewise, to satisfy the approximation $R \ll R_e + r_\pi + g_m r_\pi R_e$ we require $500\,\Omega \leq (R_e + r_\pi + \beta R_e)$, or $R_e \geq 2.4\,\Omega$. Then the power gain of the matched amplifier can be written (in dB) as:

$$G = 80 - 20\log(260 + 100R_e) \tag{3.16}$$

The power gain as a function of the series feedback resistance R_e is summarized in Table 3.1. For each value of R_e, the value of R_f that would give a simultaneous match to $50\,\Omega$ is also given.

R_e (Ω)	G (dB)	R_f (Ω)
0	31.7	1923
1	28.9	1389
2	26.7	1087
3	25.0	893
4	23.6	758
5	22.4	658
6	21.3	582
7	20.4	521
8	19.5	472
9	18.7	431
10	18.0	397

Table 3.1: Gain and shunt feedback resistance as function of series feedback resistance.

Notice that smaller values of R_e (less negative feedback) result in larger power gain. In order to satisfy the approximations that were made in the analysis it is necessary to have $R_e \geq 2.4\Omega$ and $R_f \geq 500\Omega$. From 3.1 this implies that $2.4\Omega \leq R_e \leq 7.5\Omega$, since choosing R_e to be larger than about 7.5Ω would require R_f to be smaller than 500Ω.