University of Illinois Spring 2004

ECE 361: First MidSemester Exam

Thursday February 26, 2004, 8:30 a.m. – 9:50 a.m.

This is a closed-book closed-notes examination except that one 8.5" × 11" sheet of notes is permitted: both sides may be used. Tables of integrals, calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, etc. are neither needed nor permitted.

This Examination contains three problems

Throughout this exam, an "AWGN channel" means an additive white Gaussian noise channel in which the noise process is independent of the signals being transmitted, and has two-sided power spectral density $N_0/2$ volts^2/Hz.

1. Consider a binary digital communication system operating over an AWGN channel using signals

$$s_0(t) = \begin{cases} A, & 0 \le t < T, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad s_1(t) = \begin{cases} At/T, & 0 \le t < T, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Suppose that the receiver consists of a linear filter with impulse response $h(t) = s_0(t)$ followed by a sampler at time $t = T$, and that the minimax threshold is used. What is the minimax error probability achieved by this receiver?

(b) Repeat part (a) for the case where $h(t) = s_1(t)$. The sampling time remains T and the minimax threshold appropriate for this system is used.

(c) *State* the optimum minimax error probability achievable with these signals and prove that it is smaller than your answers to parts (a) and (b).

2. **(a)** The signals $s_0(t)$ and $s_1(t)$ are used to communicate over an AWGN channel. The signal energies E_0 and E_1 respectively are such that $E_0 > E_1 > 0$. You are allowed to choose the signals subject to the above energy constraint. How would you choose the signals so as to achieve minimax error probability that is as small as possible? State clearly what this minimum error probability is.

(b) Now suppose that signals $s_2(t)$ and $s_3(t)$ of energies E_0 and E_1 respectively together with signals $s_0(t)$ and $s_1(t)$ occupy a two-dimensional signal space. Notice that even-subscripted signals have energy E_0 and odd-subscripted signals have energy E_1. Once again, you are allowed to choose signals subject to the stated energy constraint. What 4-ary signal constellation minimizes the symbol error probability? You can re-choose signals $s_0(t)$ and $s_1(t)$ if you want to in this part. *Explain* your reasoning in arriving at your answer (a formal proof that the symbol error probability is minimized is not required.)

(c) For *your* constellation of part (b), how would you assign bits b_0b_1 to the four signals so as to minimize the average bit error probability? Explain your reasoning.

(d) Now consider four additional signals $s_4(t)$ and $s_6(t)$ (of energy E_0) and $s_5(t)$ and $s_7(t)$ (of energy E_1) in the same two-dimensional space as in part (b). What 8-ary signal constellation minimizes the symbol error probability? Once again, you can re-choose signals $s_0(t)$-$s_3(t)$ if you want to in this part. *Explain* your reasoning in arriving at your answer (a formal proof that the symbol error probability is minimized is not required.)

(e) For any given value of E_1, for what value(s) of E_0 can the eight signals of part (d) be chosen to so as to lie at the intersections of uniformly spaced gridlines? Note that the signal set thus resembles a QAM signal set except that not every gridline intersection is required to have a signal point. What is the minimum distance $d_{\min}$ of this QAM-like signal set?

3. Consider a communication system that uses a 2^{2m}-ary rectangular QAM signal constellation with nearest neighbors d apart to communicate over an AWGN channel.

(a) What is the average energy per bit?

(b) What is the average symbol error probability?

(c) Now consider a receiver that operates as follows. If the sample $(\mathcal{X}_0, \mathcal{X}_1)$ is at distance $< d/2$ from a signal point $\mathbf{s}_j$, then the receiver decides that $\mathbf{s}_j$ was the transmitted signal. (Note that the sample cannot be within $d/2$ of two different signal points.) If the sample is at distance $\geq d/2$ from *all* 2^{2m} signal points, the receiver outputs a special symbol ? indicating that it is unable to make a decision. (A re-transmission is usually requested in such cases but that's a matter for another course).

Show that the probability of a *correct decision* by such a *bounded-distance* demodulator is $1 - \exp(-d^2/4N_0)$ and compare this to the probability of a correct decision by the usual optimum receiver.