University of Illinois Spring 2004

ECE 361: Second MidSemester Exam

Thursday April 15, 2004, 8:30 a.m. – 9:50 a.m.

This is a closed-book closed-notes examination except that one 8.5" × 11" sheet of notes is permitted: both sides may be used. Tables of integrals, calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, etc. are neither needed nor permitted.

This Examination contains three problems

Throughout this exam, an "AWGN channel" means an additive white Gaussian noise channel in which the noise process is independent of the signals being transmitted, and has two-sided power spectral density $N_0/2$ volts^2/Hz.

1. (40 points) An M-ary biorthogonal communication system operating over an AWGN channel transmits signals $s_{+1}, s_{-1}, s_{+2}, s_{-2}, \ldots, s_{+M/2}, s_{-M/2}$ via the respective waveforms

$$\sqrt{E_s}\psi_1(t), -\sqrt{E_s}\psi_1(t), \sqrt{E_s}\psi_2(t), -\sqrt{E_s}\psi_2(t), \ldots, \sqrt{E_s}\psi_{M/2}(t), -\sqrt{E_s}\psi_{M/2}(t),$$

where $\psi_1(t), \psi_2(t), \ldots, \psi_{M/2}(t)$ are *orthonormal* signals. The receiver has $M/2$ correlators or filters matched to the $M/2$ orthonormal signals. The receiver compares the absolute values $|\mathcal{Z}_i|$ of the decision statistics, and if $|\mathcal{Z}_j|$ is the largest, the receiver decides that s_{+j} or s_{-j} was transmitted according as $\mathcal{Z}_j > 0$ or $\mathcal{Z}_j < 0$.

In the rest of this problem, assume that s_{+1} is being transmitted, so that the decision statistics $\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_{M/2}$ are conditionally independent Gaussian random variables with means $\sqrt{E_s}, 0, \ldots, 0$ respectively and common variance $N_0/2$.

(a) Show that the (conditional) probability that the receiver decides that s_{-1} was transmitted is no larger than $Q(\sqrt{2E_s/N_0})$. Hint: What is $P\{\mathcal{Z}_1 < 0\}$?

(b) Show that the (conditional) probability that the receiver decides that s_{+j}, $j > 1$, was transmitted is bounded above by $Q(\sqrt{E_s/N_0})$.

(c) Show that the (conditional) probability that the receiver decides that s_{-j}, $j > 1$, was transmitted is also bounded above by $Q(\sqrt{E_s/N_0})$.

Now suppose further that $\mathcal{Z}_2 = -x\sqrt{N_0/2} < 0$ and that $|\mathcal{Z}_2| > \max\{|\mathcal{Z}_3|, |\mathcal{Z}_4|, \ldots, |\mathcal{Z}_{M/2}|\}$. Note the exclusion of $|\mathcal{Z}_1|$ from the above list. Thus, $|\mathcal{Z}_2|$ is not necessarily the largest of the $|\mathcal{Z}_i|$'s: in fact, $|\mathcal{Z}_1|$ might be largest, but $|\mathcal{Z}_2|$ is the "best of the rest."

(d) Explain why under the given conditions, the receiver *never* decides that $s_{\pm j}$, $j > 2$, was transmitted, or that s_{+2} was transmitted.

(e) What is the conditional probability that the receiver decides that s_{-2} was transmitted?

(f) What is the conditional probability that the receiver decides that s_{+2} was transmitted?

(g) What is the conditional probability that the receiver decides that s_{-1} was transmitted?

(h) From the answers to parts (e)-(g), deduce the conditional probability of error under the given conditions.

Note that your answers to parts (e)-(h) will involve the parameter x.

2. (40 points) An MSK signal consists of RF pulses of duration T at frequency $f_c + (4T)^{-1}$ or $f_c - (4T)^{-1}$ depending on the data bits being transmitted. Assume that f_cT is a positive integer.

(a) Ignoring all double frequency terms, show that the response of a (noncausal) narrowband filter with impulse response $h(t) = 2 \cdot \mathrm{rect}(t/T)\cos(2\pi(f_c + (4T)^{-1})t)$ to the RF pulse $\mathrm{rect}(t/T)\cos(2\pi(f_c - (4T)^{-1})t)$ is

$$\left(\frac{2T}{\pi}\right) \cdot \mathrm{rect}\left(\frac{t}{2T}\right)\cos\left(\frac{\pi t}{2T}\right)\cos(2\pi f_c t)$$

You may need the trigonometric identities

$$\begin{aligned}\cos(A \pm B) &= \cos(A)\cos(B) \mp \sin(A)\sin(B)\\ \sin(A \pm B) &= \sin(A)\cos(B) \pm \cos(A)\sin(B)\end{aligned}$$

(b) Now suppose that over $(-T/2, 3T/2)$, the input to the narrowband filter of part (a) is the PSK signal

$$\left[(-1)^{b_0}\mathrm{rect}\left(\frac{t}{T}\right) + (-1)^{b_1}\mathrm{rect}\left(\frac{t-T}{T}\right)\right]\cos\left(2\pi\left(f_c - \frac{1}{4T}\right)t\right).$$

Use linearity and superposition to show that the filter output signal during $[0, T)$ is an MSK signal.

Note: You should use the filter output stated in part (a) even if you were unable to complete the derivation of the result for yourself.

(c) How is the frequency of the output signal during $[0, T)$ related to data bits b_0 and b_1?

3. (20 points) Consider a standard noncoherent receiver that correlates the receiver input with cosine and sine signals at the carrier frequency f_c in the inphase and quadrature branches. The sample outputs at the ends of two successive signaling intervals are $(\mathcal{U}_0, \mathcal{V}_0)$ and $(\mathcal{U}_1, \mathcal{V}_1)$ respectively. Recall that for plain DPSK, the receiver decides that a 0 was transmitted if $(\mathcal{U}_1, \mathcal{V}_1)$ is closer to $(\mathcal{U}_0, \mathcal{V}_0)$ than to $(-\mathcal{U}_0, -\mathcal{V}_0)$, that is, if $\mathcal{U}_0\mathcal{U}_1 + \mathcal{V}_0\mathcal{V}_1 > 0$.

(a) Now consider a DQPSK system in which the phase *changes* of $0, \pi/2, \pi$, and $3\pi/2$ are used to send data bits 00, 01, 11, and 10. What is the decision rule for the DQPSK system? That is, how does the receiver process $(\mathcal{U}_0, \mathcal{V}_0)$ and $(\mathcal{U}_1, \mathcal{V}_1)$ to decide that whether 00, 01, 11, or 10 was transmitted? What is the decision rule for each bit in the bit pair?

(b) In a $\pi/2$-DPSK system, the value of the phase for the the signals s_0 and s_1 instantaneously increases by $\pi/2$ at the end of each signaling interval, but the modulation is nonetheless *differential;* it is the *difference* in phase between successive signaling intervals that carries the information. Thus, the phase is advanced by $\pi/2$ to send a 0 or retarded by $\pi/2$ to send a 1.

What is the decision rule for the $\pi/2$-DPSK system? That is, how does the receiver process $(\mathcal{U}_0, \mathcal{V}_0)$ and $(\mathcal{U}_1, \mathcal{V}_1)$ to decide that a 0 was transmitted?