University of Illinois at Urbana-Champaign

ECE 461: Communications II

Spring 2005
Exam I

Monday, February 28, 7:00-8:15 p.m., 165 Everitt Laboratory

Name: ____

- You have 75 minutes for this exam. The exam is closed book and closed note, except you may consult both sides of one $8.5'' \times 11''$ sheet of notes in ten point font size or larger, or equivalent handwriting size.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. __________ (10 pts.)

2. __________ (10 pts.)

3. __________ (10 pts.)

Total: __________(30 pts.)

1. Consider a baseband binary communication system with possible transmitted signals $s_1(t) = AI_{[0,T]}(t)$ and $s_0(t) = \frac{At}{T}I_{[0,T]}(t)$. Here $I_{[0,T]}(t)$ is the function that is one if $0 \leq t \leq T$ and is zero otherwise. Suppose s_1 is sent with probability $\pi_1 = \frac{1}{3}$ and s_0 is sent with probability $\pi_0 = \frac{2}{3}$. The received signal is $r(t) = s_m(t) + n(t)$, where $m \in \{0, 1\}$ and n is white Gaussian noise with two-sided power spectral density $\frac{N_o}{2}$.

(a) Sketch the Bayes optimal receiver in correlator form. Be sure to indicate the value of the threshold in the decision device.

(b) Sketch the Bayes optimal receiver using a matched filter and sampler. Be sure to indicate the filter's impulse response function.

2. Consider a binary communication system with possible transmitted signals s_1 and s_0, where

$$s_m(t) = A\left(a_m \cos(2\pi f_c t + \phi) + \beta \sin(2\pi f_c t + \phi)\right) \qquad 0 \le t \le T,\ m \in \{0, 1\}$$

where $a_1 = 1$ and $a_0 = -1$ and β is a positive constant. As usual, assume that A, T, f_c are positive constants with $f_c T >> 1$, and assume the signal is corrupted by additive white Gaussian noise with 2-sided power spectral density $\frac{N_o}{2}$. The purpose of the sine carrier term is to help the receiver recover the phase ϕ. Suppose that the receiver is able to recover the phase ϕ perfectly, and can therefore correlate the received signal over $[0, T]$ with $\psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t + \phi)$ to produce Z_1 and with $\psi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t + \phi)$ to produce Z_2.

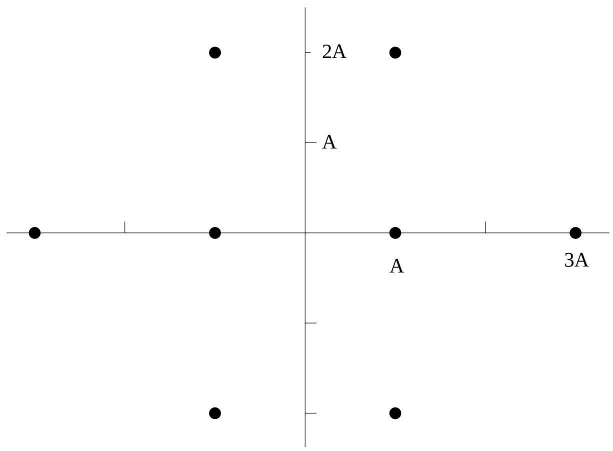
(a) Sketch the signal coordinates relative to the basis $\{\psi_1, \psi_2\}$, and sketch the maximimum likelihood decision regions.

(b) Express the probability of error in terms of A, T, N_o, and the Q function.

(c) What is the cost in dB of using the sine carrier term? In other words, if a perfect phase tracking loop could be implemented without the sine carrier term, what would be the effective difference in required energy per transmitted bit, in dB? (Hint: the answer is not zero.)

3. Consider an $8-ary$ QAM system with the following signal coordinates relative to an orthonormal basis $\{\psi_1, \psi_2\}$: $(\pm A, 0)$, $(\pm 3A, 0), (\pm A, \pm 2A)$. The signals are sent with equal probability, and are corrupted by AWGN with two-sided power spectral density $\frac{N_o}{2}$.

(a) Sketch the maximum likelihood decision regions on the following signal coordinate diagram:



(b) Express the average symbol energy and the average energy per bit in terms of A^2.

$\mathcal{E}_s =$

$\mathcal{E}_b =$

(c) Find a union bound on the probability of symbol error given that the signal with coordinates $(A, 2A)$ is transmitted. Use the minimum number of terms required.

(d) Express the maximum symbol error probability exactly in terms of A, N_o, and the Q function, for the maximum likelihood receiver. (The maximum symbol error probability is the maximum, over all eight possible transmitted symbols, of the symbol error probability.)