ECE 461 COMMUNICATIONS II **SPRING 2005**

SOLUTIONS TO FINAL EXAM

1. (28 points) *Concisely* answer each of the following seven questions.

(a) Name one advantage of using a raised cosine function as a system waveform (in comparison to a sinc waveform) and one disadvantage.

(b) Name one advantage of using the duobinary function as a system waveform (in comparison to a sinc waveform) and one disadvantage.

(c) Name one advantage of noncoherent detection over coherent detection, and one disadvantage.

(d) Explain one advantage of using an interleaver (specify the context) and one disadvantage.

(e) What is an advantage of using fractional spacing (such as $\tau = \frac{T}{2}$) in an equalizer? What is a disadvantage?

(f) What sort of modulation would you suggest for low rate communication to a satellite in deep space, and why?

(g) What sort of modulation would you suggest for high speed data communication over a one kilometer long pair of copper twisted wires (such as a short central office to home telephone line), and why?

SOLUTION

(a) The raised cosine is continuous in the frequency domain, so easier to implement than the sinc waveform, but it has a larger bandwidth.

(b) The duobinary waveform is continuous in the frequency domain, so easier to implement than the sinc waveform, but introduces (controlled) intersymbol interference.

(c) Noncoherent detection does not require phase tracking, but is less energy efficient than coherent detection when good phase tracking is possible.

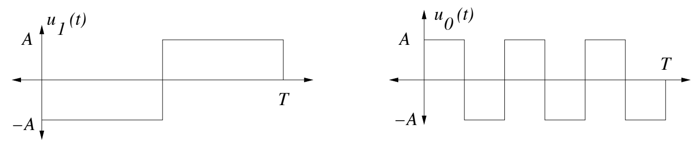
(d) Interleaving in conjunction with repetition coding (or other coding) on a fading channel yields near independence, reducing the chances that all copies of a given symbol are faded, but interleavers add complexity and delay to the system.

(e) Typically the signal bandwidth is somewhat larger than $\frac{1}{T}$ (which is the minimum possible for Nyquit pulses). Fractional spacing allows the equalizer response to be shaped over the entire signal bandwidth. The cost is increased complexity per time-width of the equalizer (i.e. either more taps are needed or the total time spread of the equalizer is reduced.)

(f) Energy is a big concern and spectral efficiency is not. An M-ary orthogonal scheme, such as M-ary pulse position modulation would be appropriate.

(g) Spectral efficiency is very important, and also energy efficiency is important. Multilevel signaling, such as QAM, should be used to get high spectral efficiency (on the order of several bits per second per Hertz). It might be used in conjunction with trellis coded modulation of orthogonal frequency division multiplexing (OFDM) to also address energy efficiency). Also, equalization or OFDM should be used to mitigate dispersion due to finite channel bandwidth.

2. (20 points) Suppose that the signal $s_m(t) = u_m(t)\cos(2\pi f_c t + \phi)$ is used to transmit a bit $m \in \{0, 1\}$, where the waveforms u_1 and u_0 are pictured. The signal is subjected to AWGN with two-sided power spectral density $\frac{N_0}{2}$. Assume that $f_c T >> 1$.



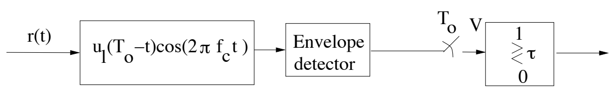
(a) Express $\frac{\mathcal{E}_b}{N_o}$ in terms of A, T, and N_o.

(b) Carefully sketch the minmax optimal receiver in matched filter form. Specify the impulse response function and threshold value. Include the carrier tracking loop, but you needn't indicate the components of the carrier tracking loop.

(c) Give the error probability for the receiver of part (b). What is the effective loss in energy efficiency, expressed in dB, for not using an antipodal pair of signals?

(d) Consider the suboptimal receiver in which the received waveform is correlated with the signal $s_1(t)$ and the correlator output is compared with a threshold. What is the minmax optimal choice of threshold in the suboptimal receiver? What is the resulting value of the probability of error? What is the total effective loss in energy efficiency, expressed in dB, for not using an ideal coherent detector and for not using an antipodal pair of signals?

(e) Consider the noncoherent receiver shown below.

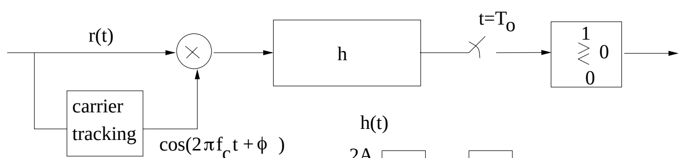


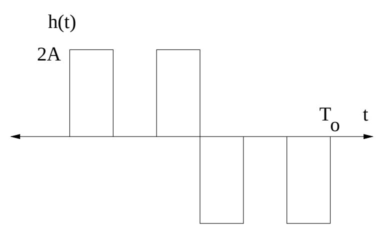
Given that $m = 1$ is sent, describe the distribution of V. Identify parameter values in terms of A,T, and N_o.

SOLUTION

(a) $\frac{\mathcal{E}_b}{N_o} = \frac{||s_m||^2}{N_o} = \frac{||u_m||^2}{2N_o} = \frac{A^2T}{2N_o}$

(b) $h(t) = u_1(T_o - t) - u_0(T_o - t)$. The signal energies are equal so the threshold is zero.





(c) $p_e = Q\left(\sqrt{\frac{||s_1-s_0||^2}{2N_o}}\right) = Q\left(\sqrt{\frac{||u_1-u_0||^2}{4N_o}}\right) = Q\left(\sqrt{\frac{2A^2T}{3N_o}}\right) = Q\left(\sqrt{\frac{2}{3}\frac{2\mathcal{E}_b}{N_o}}\right)$. The signal pair is thus $-10\log_{10}(\frac{2}{3}) = 1.8dB$ less energy efficient than an antipodal pair.

(d) The decision statistic given the transmitted bit is Gaussian. It has mean $\mu_1 = ||s_1||^2 = \frac{A^2T}{2}$ if $m = 1$, mean $\mu_0 = (s_1, s_0) = \frac{1}{2}(u_1, u_0) = -\frac{A^2T}{6}$ if $m = 0$, and variance $\sigma^2 = \frac{||s_1||^2 N_o}{2} = \frac{||u_1||^2 N_o}{4} = \frac{A^2TN_o}{4}$. The min-max threshold is $\frac{\mu_0+\mu_1}{2} = \frac{A^2T}{6}$. The corresponding probability of error is $p_e = Q(\frac{\mu_1-\mu_0}{2\sigma}) = Q\left(\sqrt{\frac{4A^2T}{9}}\right) = Q\left(\sqrt{\frac{4}{9}\frac{2\mathcal{E}_b}{N_o}}\right)$. The scheme is thus $-10\log_{10}(\frac{4}{9}) = 3.5dB$ less energy efficient than antipodal signals with ideal coherent detection.

(e) Suppose $m = 1$ is sent. Then V has the Rician distribution, with parameters v and σ^2, where $v = \frac{||u_1||^2}{2} = \frac{A^2T}{2}$ and $\sigma^2 = \frac{N_o||u_1||^2}{4} = \frac{A^2TN_o}{4}$.

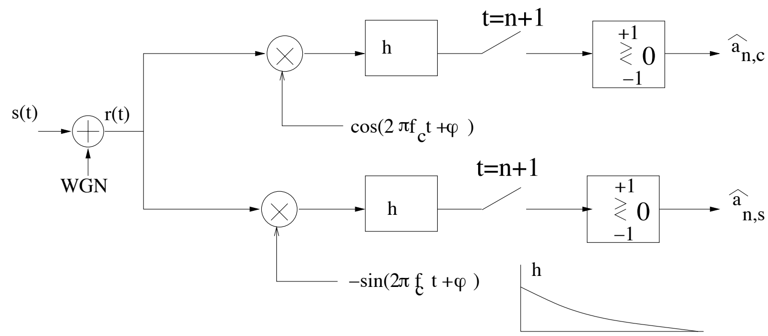
3. (28 points) Consider a QPSK system with transmitted signal

$$s(t) = \sum_n [Aa_{n,c} g_T(t-n)\cos(2\pi f_c t + \phi) - Aa_{n,s} g_T(t-n)\sin(2\pi f_c t + \phi)]$$

such that $g_T(t) = \sin(\pi t) I_{[0,1]}(t)$, the data symbols $a_{n,c}$ and $a_{n,s}$ are all in $\{1, -1\}$, and $f_c >> 1$. Note that the symbol duration is one.

(a) Sketch the signal s over the time interval $[0, 3]$, assuming whatever symbol values you'd like. Then identify the lowpass equivalent signal s_l. (Hint: It is the complex-valued baseband signal such that $s(t) = Re(s_l(t)e^{j2\pi f_c t})$. Note that this part of the problem is independent of the rest.)

Suppose the signal s is corrupted by AWGN with two-sided power spectral density $\frac{N_o}{2}$ to produce the received signal r, which is fed into the receiver indicated.



The receive filter h is given by $h(t) = e^{-t}$ for $t \geq 0$ and $h(t) = 0$ for $t < 0$, and perfect phase tracking is assumed.

(b) Ignoring the effects of intersymbol interference, find the average probability of error. Roughly how many dB loss is there compared to that of the ideal matched filter receiver? Hint: $\int_0^1 e^{-t}\sin(\pi t)dt \approx 0.40$.

For the remainder of this problem, the effects of intersymbol interference are to be taken into account. The block diagram is repeated for convenience.

(c) Do the bits in the quadraphase signal interfere with the bits in the inphase signal? Explain.

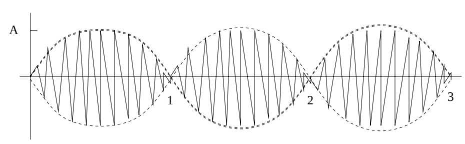
(d) Identify the worst case bit sequence for receiving bit $a_{0,c}$, and find the corresponding maximum error probability. (Hint: $e^{-1} \approx 0.38$.)

(e) Suppose a three-tap equalizer is inserted after the filter in each branch of the receiver, and that the vector of tap weights (c_{-1}, c_0, c_1) is chosen to minimize the peak relative distortion. Find the tap weights.

(f) Repeat part (d), but now with the equalizer installed. (Hint: Account for the effect of the equalizer on the noise.)

(g) Of the answers to parts (b), (d), and (f), which is the largest error probability? Which is smallest?

SOUTION (a)



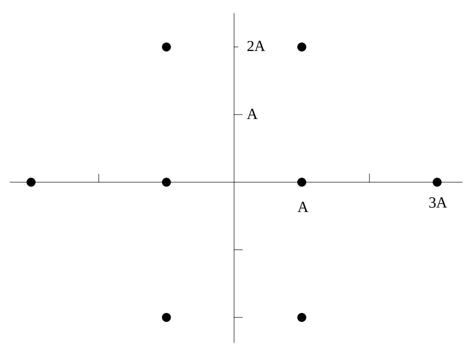
$s_l(t) = Ae^{j\phi}\sum_n (a_{n,c} + ja_{n,s})g_T(t-n)$

(b) If, for example, $a_{0,c} = 1$, the decision statistic is normal with mean $\mu = \frac{A}{2}h * g_T(1) \approx \frac{(0.4)A}{2} = (0.2)A$ and variance $\sigma^2 = \frac{N_o||h||^2}{4} = \frac{N_o}{8}$. Thus, the probability of error is $p_e = Q(\frac{\mu}{\sigma}) = Q\left(\sqrt{\frac{A^2(0.32)}{N_o}}\right)$. Since $\frac{2\mathcal{E}_b}{N_o} = \frac{A^2}{N_o}$, the loss versus the optimal receiver is $-10\log_{10}(0.32)dB \approx 5dB$.

(c) No, because the carrier tracking is perfect, the quadraphase signal does not pass the filter in the

upper branch of the receiver.
(d) The sequence $a_{c,0} = 1$ and $a_{c,n} = -1$ for $n < 0$ would be the worst, because the previous bits would all subtract from the sample for $a_{c,0}$. Since $h * s(n) = h * s(1)e^{-(n-1)} \approx (0.4)e^{-(n-1)}$ for $n \geq 1$, the peak relative distortion is $d^* = \frac{e^{-1}+e^{-2}+e^{-3}+\cdots}{1} = \frac{e^{-1}}{1-e^{-1}} \approx 0.6$. In the worst case, the effect of ISI on the bit error probability is to multiply $\frac{\mathcal{E}_b}{N_o}$ by $(1-d^*)^2$. Therefore, the error probability is $p_e = Q\left(\sqrt{\frac{A^2(0.32)(0.6)^2}{N_o}}\right)$, or an energy cost of $-10\log_{10} 0.6^2 = 4.4dB$ compared to part (c), or 9.4 dB compared to an optimal detector with no ISI present.
(e) The nonzero values of $h*g_E(n)$ are $(c_{-1}, c_0+c_{-1}/e, c_1+c_0/e+c_{-1}/e^2, \ldots)$. Selecting $c = (0, 1, -e^{-1})$ (or $c = (1, -e^{-1}, 0)$) completely eliminates the ISI.
(f) There is no residual ISI, so the error probability is the same for any bit pattern. The new noise variance is $\sigma^2(1+|e^{-1}|^2) = 1.13\sigma^2$, so that the error probability is given by $p_e = Q\left(\sqrt{\frac{A^2(0.32)}{(1.13)N_o}}\right)$, or an energy cost of $-10\log_{10} 1.13 = 0.55dB$ compared to part (b), or 5.55 dB compared to an optimal detector.
(g) The error for part (b) is the smallest and the error for part (d) is the largest. (The ISI causes a loss of 4.4 dB (part (d)), whereas most of that is made up for by using the equalizer.)

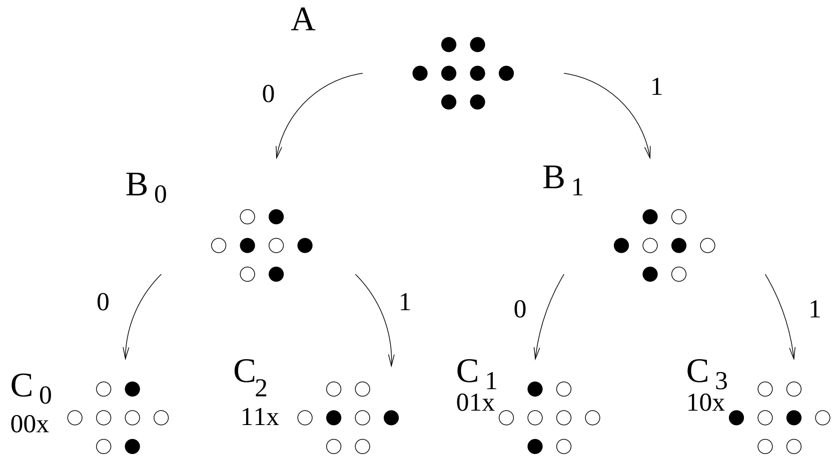
4. (24 points) This problem focuses on using trellis coded modulation using the following 8-ary QAM constellation, where the coordinates are taken relative to an orthonormal basis for the signal space.



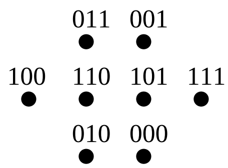
Suppose the $(n = 2, k = 1)$ convolutional code with generators $\mathbf{g}_1 = [1\ 0\ 1]$ and $\mathbf{g}_2 = [1\ 1\ 1]$ is used to produce two coded bits per data bit to select one of 4 subconstellations, and then an additional uncoded data bit is used to specify a symbol within the selected subconstellation. This scheme has the same bandwidth requirement as an uncoded QPSK system.
(a) Express the energy per data bit, $\mathcal{E}_b$, in terms of A. (Remember that there are two data bits per symbol.)
(b) Apply the method of set partitioning to the signal constellation. As in the text, label the original constellation A, the two subconstellations below it B_0 and B_1, and the four constellations below those, C_0, C_2, C_1, and C_3.
(c) Show a labeling of the signal constellation, following the design guidelines given in the text.
(d) Sketch three stages of the trellis, and indicate the corresponding transmitted symbols on the edges. Your answer should lead to the largest possible Euclidean distance for deviating paths of length one and length three. (Two templates are given for convenience. If one is used only for scratch, cross it out.)
(e) What is the minimum free Euclidean distance squared for a deviating path of length one edge? Of length three edges? Of length four edges? (Express your three answers in terms of A^2)
(f) Using the answers to parts (a) and (e), give a three term approximation to the bit error probability (e.g. the first three terms of the union bound) in terms of the Q function and $\frac{\mathcal{E}_b}{N_o}$. What is the approximate energy efficiency gain over uncoded QPSK (which has the same spectral efficiency)?

SOLUTION
(a) The average energy per symbol is given by $2\mathcal{E}_b = \frac{2A^2+2(3A)^2+4(1^2+2^2)A^2}{8} = 5A^2$, so $\mathcal{E}_b = \frac{5A^2}{2}$.

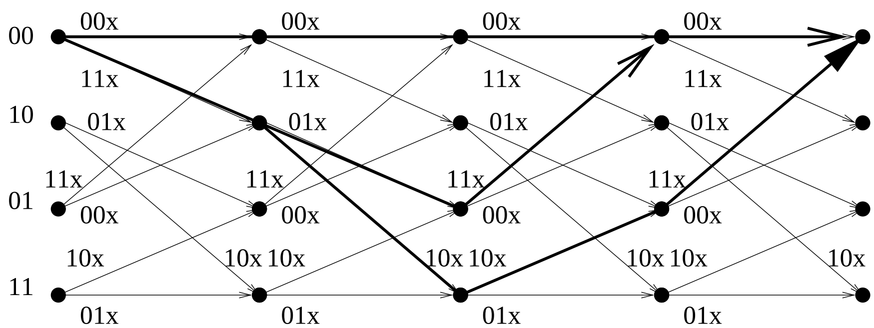
(b)



(c)



(d)



(e) Single edge deviations are parallel edges in the same C_i, and have squared Euclidean distance $(4A)^2 = 16A^2$.
The Euclidean distance for a length three deviation is $(2\sqrt{2}A)^2 + (2A)^2 + (2\sqrt{2}A)^2 = 20A^2$, because the first and third symbols are in the same B-level constellation as the true path, and the symbol for the middle edge has the minimum distance, $2A$.
The Euclidean distance for a length four deviation is $(2\sqrt{2}A)^2 + (2A)^2 + (2A)^2 + (2\sqrt{2}A)^2 = 24A^2$, because the first and fourth symbols are in the same B-level constellation as the true path, and the symbols for the two middle edges have the minimum distance, $2A$.
(f) Opportunities for length one, three, or four deviating paths happen once per symbol. There is one bit error for a length one or length three deviation, and two bit errors for a length four deviation. There are two bits per symbol. This suggests the approximation for the bit error probability:

$$
\begin{aligned}
p_e &\approx \frac{1}{2}\left[Q\left(\sqrt{\frac{16A^2}{2N_o}}\right) + Q\left(\sqrt{\frac{20A^2}{2N_o}}\right) + 2Q\left(\sqrt{\frac{24A^2}{2N_o}}\right)\right] \\
&= \frac{1}{2}\left[Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_o}1.6}\right) + Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_o}2.0}\right) + 2Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_o}2.4}\right)\right]
\end{aligned}
$$

The predicted energy efficiency over uncoded QPSK is thus $10\log_{10}(1.6) \approx 2dB$. (Note: Interestingly enough, for uncoded systems, the constellation of this problem is more energy efficient that 8-ary PSK. However, for use with trellis coding, the 8-ary PSK constellation is better, at least for high $\frac{\mathcal{E}_b}{N_o}$.)