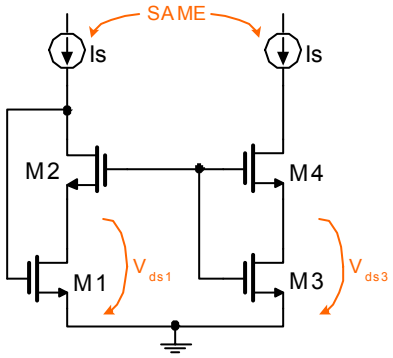
High-Swing Cascode Biasing

Assume M1, M2, M4 are in saturation, M3 is in triode region.
Vds1 is set to 50~100mV above Vgt1 by controlling Vds3.
Assume square law.

Sizing:
M1 --> $\beta = \mu\, Cox\, (W/L)$, $Vgt = Vgs - Vth$
M2, M4 --> βc, $Vgtc$
M3 --> β/n, Vgt'



Cascode Biasing

KVL: $V_{gs2} + V_{ds1} = V_{gs4} + V_{ds3} = V_{gs3}$

$$I_s = \frac{1}{2}\cdot\beta\cdot V_{gt}^{\,2} = \frac{1}{2}\cdot\beta_c\cdot V_{gtc}^{\,2} = \frac{\beta}{n}\cdot\left(V_{gt}'\cdot V_{ds3} - \frac{1}{2}\cdot V_{ds3}^{\,2}\right)$$

If M2 and M4 are same size matched devices, Vgs2 = Vgs4, so we have,

$$V_{ds1} = V_{ds3}$$

Also, $V_{ds3} + V_{gs4} = V_{gs3}$ --> $V_{gt}' = V_{gtc} + V_{ds3} + \left(V_{thc} - V_{th}\right) = V_{gtc} + V_{ds3} + \Delta V_{th}$

ΔVth is the threshold voltage difference between the cascode (M4) and bottom device (M3)

So, $$\frac{\beta}{n}\cdot\left[\left(\sqrt{\frac{\beta}{\beta_c}}\cdot V_{gt} + V_{ds1} + \Delta V_{th}\right)\cdot V_{ds1} - \frac{1}{2}\cdot V_{ds1}^{\,2}\right] = \frac{1}{2}\cdot\beta\cdot V_{gt}^{\,2}$$

Let, $$\gamma = \sqrt{\frac{\beta}{\beta_c}} + \frac{\Delta V_{th}}{V_{gt}}$$

So, $$\frac{\beta}{n}\cdot\left[\left(\gamma\cdot V_{gt} + V_{ds1}\right)\cdot V_{ds1} - \frac{1}{2}\cdot V_{ds1}^{\,2}\right] = \frac{1}{2}\cdot\beta\cdot V_{gt}^{\,2}$$

Solve, obtain $$V_{ds1} = V_{ds3} = \left(-\gamma + \sqrt{\gamma^2 + n}\right)\cdot V_{gt} = K\cdot V_{gt}$$

Conclusion

If $\frac{\Delta V_{th}}{V_{gt}}$ is small compared to $\sqrt{\frac{\beta}{\beta_c}}$, then $K = \left(-\gamma + \sqrt{\gamma^2 + n}\right)$ largely determined by sizing only.
Vds1 will track Vgt.

e.x., let K = 1.25, we obtain n = (5K+6.25)/4. Usually, set n = 3 or 4 is good enough for safe biasing of cascode transistors.