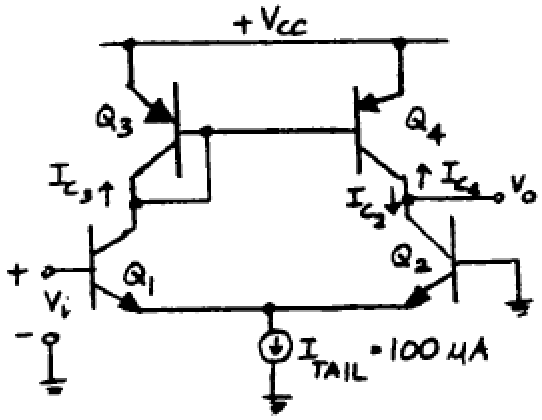
4.12

$V_{A_{pnp}} = 50\,V \qquad V_{A_{npn}} = 130\,V$

$\beta_{F_{pnp}} = 50 \qquad \beta_{F_{npn}} = 200$

Assume no device mismatch



$I_{C_1} \approx I_{C_2} = \frac{I_{EE}}{2} = 50\,\mu A = I_C$

$I_{C_3} \approx I_{C_4} = -50\,\mu A$

$\therefore g_m = \frac{I_C}{V_T} = \frac{50\,\mu A}{26\,mV} = \frac{1}{520}\,℧$

$r_{o_{pnp}} = \frac{V_{A_{pnp}}}{I_C} = 1\,M\Omega$

$r_{o_{npn}} = \frac{V_{A_{npn}}}{I_C} = 2.6\,M\Omega$

Since unloaded, the output resistance is

$R_{OUT} = r_{o_{pnp}} \parallel r_{o_{npn}} = 722\,K\Omega$

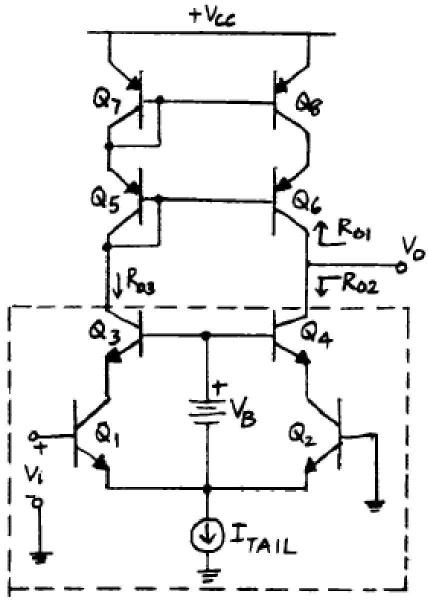
$A_V = g_m R_{OUT} = \frac{1}{520} \times 722\,K$

$= 1388$

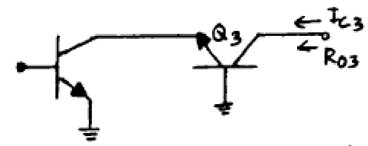
```
BJT DIFFERENTIAL PAIR WITH CURRENT-MIRROR LOAD
******
VCC   100 0   2.5
VEE   200 0   -2.5
Q1    2   3   4    NPN
Q2    6   0   4    NPN
Q3    2   2   100  PNP
Q4    6   2   100  PNP
ITAIL 4   200 100U
RTAIL 4   200 1MEG
.MODEL NPN NPN RB=200 BF=200 VAF=130 IS=5E-15
.MODEL PNP PNP RB=300 BF=50  VAF=50  IS=2E-15

* A DC INPUT VOLTAGE OF ABOUT 1 MV FORCES THE COLLECTOR-EMITTER
* VOLTAGE OF Q2 ABOUT EQUAL TO THAT OF Q1.
VIDC  3   5   1M
VI    5   0
.OPTIONS NOMOD NOPAGE
.WIDTH OUT=80
.OP
.DC VI -0.01 0.01 0.002
.PLOT DC V(6)
.TF V(6) VI
.END

 ******  DC TRANSFER CURVES       TNOM=  27.000 TEMP=  27.000

    VOLT       V(6)
(A        )     -1.000E-00      0.          1.000E-00     2.000E+00    3.000E+00
                     +            +            +            +            +
-1.000E-02 -5.47E-01-+------A+-----+------+------+------+------+------+------+-
-8.000E-03 -5.39E-01 +      A+     +      +      +      +      +      +      +
-6.000E-03 -5.29E-01 +      A      +      +      +      +      +      +      +
-4.000E-03 -5.12E-01 +      A      +      +      +      +      +      +      +
-2.000E-03 -4.63E-01 +      +A     +      +      +      +      +      +      +
 0.         1.85E+00 +      +      +      +      +      +    A +      +      +
 2.000E-03  2.39E+00 +      +      +      +      +      +      +     A+      +
 4.000E-03  2.41E+00 +      +      +      +      +      +      +     A+      +
 6.000E-03  2.42E+00 +      +      +      +      +      +      +     A+      +
 8.000E-03  2.43E+00 +      +      +      +      +      +      +     A+      +
 1.000E-02  2.44E+00-+------+------+------+------+------+------+-----A+------+-
                     +            +            +            +            +
 ****  OPERATING POINT INFORMATION      TNOM=  27.000 TEMP=  27.000
   NODE      =VOLTAGE      NODE      =VOLTAGE      NODE      =VOLTAGE
+0:2         = 1.880E+00 0:3         = 1.000E-03 0:4         =-5.951E-01
+0:5         =    0.     0:6         = 1.857E+00 0:100       = 2.500E+00
+0:200       =-2.500E+00

**** BIPOLAR JUNCTION TRANSISTORS
SUBCKT
ELEMENT  0:Q1        0:Q2        0:Q3        0:Q4
MODEL    0:NPN       0:NPN       0:PNP       0:PNP
 IB       2.547E-07   2.451E-07  -9.939E-07  -9.939E-07
 IC       5.168E-05   4.972E-05  -4.970E-05  -4.972E-05
 VBE      5.961E-01   5.951E-01  -6.194E-01  -6.194E-01
 VCE      2.475E+00   2.452E+00  -6.194E-01  -6.426E-01
 VBC     -1.879E+00  -1.857E+00   0.          2.318E-02
 VS      -1.880E+00  -1.857E+00  -1.880E+00  -1.880E+00
 POWER    1.281E-04   1.221E-04   3.140E-05   3.256E-05
 BETAD    2.028E+02   2.028E+02   5.000E+01   5.002E+01
 GM       1.998E-03   1.922E-03   1.920E-03   1.921E-03
 RPI      1.015E+05   1.055E+05   2.602E+04   2.602E+04
 RX       2.000E+02   2.000E+02   3.000E+02   3.000E+02
 RO       2.551E+06   2.652E+06   1.006E+06   1.006E+06
 BETAAC   2.028E+02   2.028E+02   4.997E+01   4.999E+01

****        SMALL-SIGNAL TRANSFER CHARACTERISTICS

     V(6)/VI                                   =  1.398E+03
     INPUT RESISTANCE AT          VI           =  2.853E+05
     OUTPUT RESISTANCE AT V(6)                 =  7.294E+05
```

```
BJT DIFFERENTIAL PAIR WITH CURRENT-MIRROR LOAD
* (COMMON-MODE INPUT)
******
VCC   100 0   2.5
VEE   200 0   -2.5
Q1    2   3   4    NPN
Q2    6   5   4    NPN
* |CMRR| = 1398/0.0002561 = 5.46E6
Q3    2   2   100  PNP
Q4    6   2   100  PNP
ITAIL 4   200 100U
RTAIL 4   200 1MEG
.MODEL NPN NPN RB=200 BF=200 VAF=130 IS=5E-15
.MODEL PNP PNP RB=300 BF=50  VAF=50  IS=2E-15

* A DC INPUT VOLTAGE OF ABOUT 1 MV FORCES THE COLLECTOR-EMITTER
* VOLTAGE OF Q2 ABOUT EQUAL TO THAT OF Q1.
VIDC  3   5   1M
VI    5   0
.OPTIONS NOMOD NOPAGE
.WIDTH OUT=80
.OP
.DC VI -0.01 0.01 0.002
.PLOT DC V(6)
.TF V(6) VI
.END

 ******  DC TRANSFER CURVES       TNOM=  27.000 TEMP=  27.000
******

    VOLT       V(6)
(A        )     1.857E+00     1.857E+00    1.857E+00    1.857E+00    1.857E+00
                     +            +            +            +            +
-1.000E-02  1.85E+00-+------+------+------+------+------+------+--A---+------+-
-8.000E-03  1.85E+00 +      +      +      +      +      +     A+      +      +
-6.000E-03  1.85E+00 +      +      +      +      +      + A    +      +      +
-4.000E-03  1.85E+00 +      +      +      +      +     A+      +      +      +
-2.000E-03  1.85E+00 +      +      +      +      + A    +      +      +      +
 0.         1.85E+00 +      +      +      +     A+      +      +      +      +
 2.000E-03  1.85E+00 +      +      +      + A    +      +      +      +      +
 4.000E-03  1.85E+00 +      +      +     A+      +      +      +      +      +
 6.000E-03  1.85E+00 +      +      + A    +      +      +      +      +      +
 8.000E-03  1.85E+00 +      +     A+      +      +      +      +      +      +
 1.000E-02  1.85E+00-+------+-A----+------+------+------+------+------+------+-
                     +            +            +            +            +
 ****  OPERATING POINT INFORMATION      TNOM=  27.000 TEMP=  27.000
   NODE      =VOLTAGE      NODE      =VOLTAGE      NODE      =VOLTAGE
+0:2         = 1.880E+00 0:3         = 1.000E-03 0:4         =-5.951E-01
+0:5         =    0.     0:6         = 1.857E+00 0:100       = 2.500E+00
+0:200       =-2.500E+00

**** BIPOLAR JUNCTION TRANSISTORS
SUBCKT
ELEMENT  0:Q1        0:Q2        0:Q3        0:Q4
MODEL    0:NPN       0:NPN       0:PNP       0:PNP
 IB       2.547E-07   2.451E-07  -9.939E-07  -9.939E-07
 IC       5.168E-05   4.972E-05  -4.970E-05  -4.972E-05
 VBE      5.961E-01   5.951E-01  -6.194E-01  -6.194E-01
 VCE      2.475E+00   2.452E+00  -6.194E-01  -6.426E-01
 VBC     -1.879E+00  -1.857E+00   0.          2.318E-02
 VS      -1.880E+00  -1.857E+00  -1.880E+00  -1.880E+00
 POWER    1.281E-04   1.221E-04   3.140E-05   3.256E-05
 BETAD    2.028E+02   2.028E+02   5.000E+01   5.002E+01
 GM       1.998E-03   1.922E-03   1.920E-03   1.921E-03
 RPI      1.015E+05   1.055E+05   2.602E+04   2.602E+04
 RX       2.000E+02   2.000E+02   3.000E+02   3.000E+02
 RO       2.551E+06   2.652E+06   1.006E+06   1.006E+06
 BETAAC   2.028E+02   2.028E+02   4.997E+01   4.999E+01

****        SMALL-SIGNAL TRANSFER CHARACTERISTICS

     V(6)/VI                                   = -2.561E-04
     INPUT RESISTANCE AT          VI           =  1.152E+08
     OUTPUT RESISTANCE AT V(6)                 =  7.294E+05
```

4.16

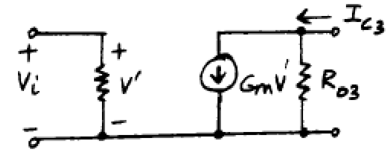


Half circuit of the dotted box :



is the CE-CB configuration.

∴ the equivalent circuit is



Where $R_i = r_{\pi_1}$, $G_m = g_{m_1}$

$$R_{o3} = \beta_3 r_{o3} = \beta_4 r_{o4} = R_{o2}$$

$$R_{OUT} = R_{o1} \parallel R_{o2}$$

$$R_{o1} \approx \frac{\beta_6 r_{o6}}{2}$$

$$I_{c_1} \approx I_{c_2} \approx I_{c_3} = I_{c_4} = -I_{c_5} \approx -I_{c_6} \approx -I_{c_7} = -I_{c_8} \approx I_{EE}/2$$

$$\therefore g_{m_1} = g_{m_2} = g_{m_3} = g_{m_4} = g_{m_5} = g_{m_6} = g_{m_7} = g_{m_8} = g_m$$

$$\beta_4 = \beta_{npn} = 200 \,, \quad \beta_6 = \beta_{pnp} = 50$$

$$\therefore r_{o6} = \frac{1}{g_m \eta_{pnp}} \,, \quad r_{o4} = \frac{1}{g_m \eta_{npn}}$$

$$R_{OUT} = R_{o1} \parallel R_{o2} = \frac{1}{\frac{2 g_m \eta_{pnp}}{\beta_6} + \frac{g_m \eta_{npn}}{\beta_4}}$$

$$A_V = g_m R_{OUT} = \frac{1}{\frac{2\eta_{pnp}}{\beta_6} + \frac{\eta_{npn}}{\beta_4}}$$

$$A_V = \left[1 \Big/ \left(2 \times \frac{5 \times 10^{-4}}{50} + \frac{2 \times 10^{-4}}{200} \right) \right] = 4.8 \times 10^{4}$$

```
BJT CASCODED DIFFERENTIAL PAIR WITH CASCODED CURRENT-MIRROR LOAD
VCC   100 0   2.5
VEE   200 0   -2.5
VBIAS 9   2   1.3
Q1    3   1   2    NPN
Q2    4   0   2    NPN
Q3    5   9   3    NPN
Q4    6   9   4    NPN
Q5    5   5   7    PNP
Q6    6   5   8    PNP
Q7    7   7   100  PNP
Q8    8   7   100  PNP
ITAIL 2   200 100U
RTAIL 2   200 1MEG
.MODEL NPN NPN RB=200 BF=200 VAF=130 IS=5E-15
.MODEL PNP PNP RB=300 BF=50  VAF=50  IS=2E-15

* A DC INPUT VOLTAGE OF ABOUT 2 MV FORCES THE VOLTAGE FROM
* THE OUTPUT TO GROUND TO BE ABOUT EQUAL TO THE VOLTAGE
* FROM NODE 5 (THE CASCODE CURRENT MIRROR INPUT) TO GROUND.
VIDC  1   10  2M
VI    10  0
.OPTIONS NOMOD NOPAGE
.WIDTH OUT=80
.OP
.DC VIDC -0.01 0.01 0.001
.PLOT DC V(6)
.TF V(6) VI
.END

****** DC TRANSFER CURVES      TNOM=  27.000 TEMP=  27.000
   VOLT        V(6)
(A      )       0.          5.000E-01    1.000E+00    1.500E+00    2.000E+00
                     +            +            +            +            +
-1.000E-02  1.06E-01-+--A---+-----+------+------+------+------+------+------+-
-9.000E-03  1.11E-01 +  A   +     +      +      +      +      +      +      +
-8.000E-03  1.15E-01 +  A   +     +      +      +      +      +      +      +
-7.000E-03  1.20E-01 +  A   +     +      +      +      +      +      +      +
-6.000E-03  1.25E-01 +   A  +     +      +      +      +      +      +      +
-5.000E-03  1.31E-01 +   A  +     +      +      +      +      +      +      +
-4.000E-03  1.36E-01 +   A  +     +      +      +      +      +      +      +
-3.000E-03  1.43E-01 +   A  +     +      +      +      +      +      +      +
-2.000E-03  1.51E-01 +   A  +     +      +      +      +      +      +      +
-1.000E-03  1.60E-01 +   A  +     +      +      +      +      +      +      +
 0.         1.73E-01-+----A-+-----+------+------+------+------+------+------+-
 1.000E-03  1.93E-01 +    A +     +      +      +      +      +      +      +
 2.000E-03  5.86E-01 +      +     + A    +      +      +      +      +      +
 3.000E-03  1.77E+00 +      +     +      +      +      +      +      +A     +
 4.000E-03  1.79E+00 +      +     +      +      +      +      +      +A     +
 5.000E-03  1.80E+00 +      +     +      +      +      +      +      + A    +
 6.000E-03  1.81E+00 +      +     +      +      +      +      +      + A    +
 7.000E-03  1.82E+00 +      +     +      +      +      +      +      + A    +
 8.000E-03  1.82E+00 +      +     +      +      +      +      +      + A    +
 9.000E-03  1.83E+00 +      +     +      +      +      +      +      + A    +
 1.000E-02  1.83E+00-+------+-----+------+------+------+------+------+-A----+-
                     +            +            +            +            +

**** OPERATING POINT INFORMATION      TNOM=  27.000 TEMP=  27.000
  NODE     =VOLTAGE        NODE     =VOLTAGE        NODE     =VOLTAGE
+0:1       = 2.000E-03 0:2          =-5.950E-01 0:3          = 1.081E-01
+0:4       = 1.100E-01 0:5          = 1.260E+00 0:6          = 5.857E-01
+0:7       = 1.880E+00 0:8          = 1.879E+00 0:9          = 7.050E-01
+0:10      =   0.      0:100        = 2.500E+00 0:200        =-2.500E+00

**** BIPOLAR JUNCTION TRANSISTORS
SUBCKT
ELEMENT  0:Q1        0:Q2        0:Q3        0:Q4
MODEL    0:NPN       0:NPN       0:NPN       0:NPN
 IB       2.644E-07   2.447E-07   2.621E-07   2.439E-07
 IC       5.292E-05   4.899E-05   5.265E-05   4.874E-05
 VBE      5.970E-01   5.950E-01   5.968E-01   5.950E-01
 VCE      7.032E-01   7.050E-01   1.152E+00   4.757E-01
 VBC     -1.061E-01  -1.100E-01  -5.558E-01   1.192E-01
 VS      -1.081E-01  -1.100E-01  -1.260E+00  -5.857E-01
 POWER    3.737E-05   3.468E-05   6.084E-05   2.333E-05
 BETAD    2.001E+02   2.001E+02   2.008E+02   1.998E+02
 GM       2.045E-03   1.894E-03   2.035E-03   1.884E-03
 RPI      9.783E+04   1.056E+05   9.866E+04   1.060E+05
 RX       2.000E+02   2.000E+02   2.000E+02   2.000E+02
 RO       2.458E+06   2.656E+06   2.479E+06   2.664E+06
 BETAAC   2.001E+02   2.001E+02   2.008E+02   1.997E+02

SUBCKT
ELEMENT  0:Q5        0:Q6        0:Q7        0:Q8
MODEL    0:PNP       0:PNP       0:PNP       0:PNP
 IB      -1.014E-06  -9.619E-07  -9.941E-07  -9.941E-07
 IC      -5.068E-05  -4.874E-05  -4.970E-05  -4.970E-05
 VBE     -6.199E-01  -6.185E-01  -6.194E-01  -6.194E-01
 VCE     -6.199E-01  -1.293E+00  -6.194E-01  -6.208E-01
 VBC       0.         6.750E-01    0.         1.369E-03
 VS      -1.261E+00  -1.261E+00  -1.880E+00  -1.880E+00
 POWER    3.204E-05   6.364E-05   3.140E-05   3.147E-05
 BETAD    5.000E+01   5.067E+01   5.000E+01   5.000E+01
 GM       1.958E-03   1.884E-03   1.921E-03   1.921E-03
 RPI      2.551E+04   2.689E+04   2.601E+04   2.601E+04
 RX       3.000E+02   3.000E+02   3.000E+02   3.000E+02
 RO       9.866E+05   1.039E+06   1.006E+06   1.006E+06
 BETAAC   4.997E+01   5.064E+01   4.997E+01   4.997E+01

****     SMALL-SIGNAL TRANSFER CHARACTERISTICS

    V(6)/VI                                    =  4.909E+04
    INPUT RESISTANCE AT          VI            =  2.148E+05
    OUTPUT RESISTANCE AT V(6)                  =  2.611E+07
```

4.22

circuit as shown in fig (4.31a)

with $V_{CC} = 15\,V$, $R_1 = 20\,k\Omega$, $R_2 = 10\,k\Omega$

$$I_{IN} = \frac{15-0.7}{20\,k\Omega} = 0.72\ mA$$

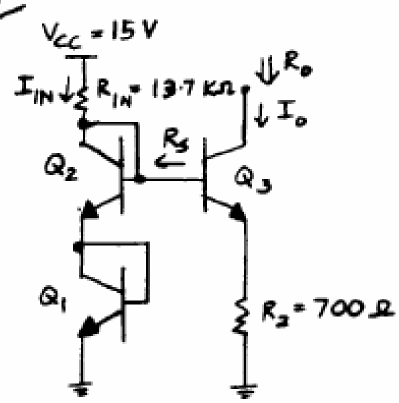
$$I_{OUT} R_2 = V_T \ln \frac{I_{C1}}{I_{OUT}} \approx V_T \ln \frac{I_{IN}}{I_{OUT}}$$

$$I_{OUT} = \frac{V_T}{R_2} \ln \frac{I_{IN}}{I_{OUT}} = 2.6 \times 10^{-6} \ln \frac{0.72\,mA}{I_{OUT}}$$

By trial and error we find out,

$$I_{OUT} \doteqdot 10.9\ \mu A$$

4.25



Assume:

$I_S = 5\times 10^{-15} A$

$\beta_o = 200$

$$V_{BE_1} + V_{BE_2} = V_{BE_3} + I_o R_2$$

$$I_{C_1} \simeq I_{C_2} = I_{IN} \;;\; I_o = I_{E_3}$$

$$I_{IN} = \frac{15 - 2\times 0.7}{13.7\,k} = 0.993\,mA$$

$$V_T \ln \frac{I_{C_1}}{I_S} + V_T \ln \frac{I_{C_2}}{I_S} = V_T \ln \frac{I_o}{I_S} + I_o R_2$$

$$\therefore I_o = \frac{V_T}{R_2} \ln \frac{I_{IN}^2}{I_o I_S} = \frac{26mV}{700\Omega} \ln \frac{(0.993\times 10^{-3})^2}{(5\times 10^{-15} I_o)}$$

By trial and error, $I_o \simeq 0.97\,mA$

R_S is the equivalent resistance looking into the base of Q_2

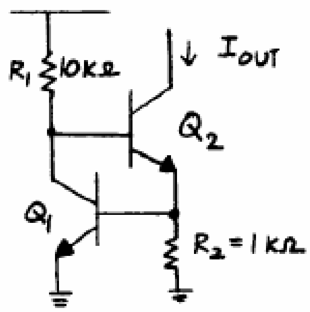
$$R_S \simeq \frac{2}{g_m} = 2\frac{V_T}{I_{IN}} \simeq 52\,\Omega \ll r_{\pi_3}$$

where $r_{\pi_3} \simeq 2.6\,k\Omega \;\therefore\; g_{m_3} = \frac{1}{26}\,\mho$

$$R_o = r_{o3}\left(1 + \frac{g_{m_3} R_2}{1 + \frac{g_{m_3} R_2}{\beta_o}}\right) = 3.2\,M\Omega$$

where, $r_{o3} = \frac{V_A}{I_o} \simeq 130\,k\Omega$

4.26



$$S = \frac{V_{CC}}{I_{OUT}} \frac{\partial I_{OUT}}{\partial V_{CC}}$$

For this circuit

$$I_{OUT} \simeq \frac{V_{BE_1}}{R_2}$$

$$I_{C_1} = \frac{V_{CC} - V_{BE_1} - V_{BE_2}}{R_1} \quad \text{and}$$

$$V_{BE_1} = V_T \ln \frac{I_{C_1}}{I_S}$$

$$\therefore I_{OUT} = \frac{V_T}{R_2} \ln \frac{V_{CC} - V_{BE_1} - V_{BE_2}}{I_S R_1}$$

If $V_{BE(on)} << V_{CC}$ then,

$$\frac{\partial I_{OUT}}{\partial V_{CC}} \approx \frac{V_T}{R_2} \frac{1}{V_{CC} - V_{BE_1} - V_{BE_2}}$$

and $S = \dfrac{V_{CC}}{I_{OUT}} \dfrac{V_T}{R_2} \dfrac{1}{V_{CC} - V_{BE_1} - V_{BE_2}}$

$$I_{C_1} \approx \frac{15 - 0.7 - 0.7}{10} = 1.36 \text{ mA}$$

$$\therefore V_{BE_1} \cong 26 \ln \frac{1.36 \times 10^{-3}}{5 \times 10^{-5}} \text{ mV}$$

$$= 0.685 \text{ V}$$

$$\therefore I_{OUT} = \frac{0.685}{1} = 0.685 \text{ mA}$$

$$\therefore S = \frac{15}{0.685 \times 10^{-3}} \frac{26 \times 10^{-3}}{1000} \frac{1}{13.6} = 0.04$$