ECE 489 Robot Dynamics and Control

Homework # 3: Stability

Due Date: Tuesday, February 27, 2007

1. Let S be an $n \times n$ skew-symmetric matrix. Prove that $x^T S x = 0$ for all $x \in \Re^n$.

2. Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_1 - x_1 x_2 \\ \dot{x}_2 &= 2x_1^2 - 2x_2 \end{aligned}$$

   Find the equilibrium points and investigate local stability around each equilibrium point.

3. Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_1 x_2^2 \\ \dot{x}_2 &= -x_2 - x_2 x_1^2 \end{aligned}$$

   Show that $(0,0)$ is the unique equilibrium point and investigate local stability. Investigate global stability using the Lyapunov function candidate

$$V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$$

4. Consider the nonlinear system

$$\dot{x} = f(x) + g(x)u$$

   with $f(0) = 0$. Suppose there exists a positive definite $V(x)$, with $V(0) = 0$, such that

$$\frac{\partial V(x)}{\partial x} f(x) \leq 0$$

   Show that the origin is asymptotically stable using the control law

$$u = -\frac{\partial V(x)}{\partial x} g(x)$$

   provided

$$\frac{\partial V(x)}{\partial x} g(x) \neq 0$$

   at all points where

$$\frac{\partial V(x)}{\partial x} f(x) = 0$$

   The control law u so defined is called a *Jurdjevic-Quinn* or $L_g V$ control.