ECE 489 Homework # 1

Solutions

2-2 Notice that $||v||^2 = v^T v \Rightarrow ||v|| = +\sqrt{v^T v}$. Therefore,

$$\begin{aligned} ||Rv|| &= +\sqrt{(Rv)^T Rv} = \sqrt{v^T R^T Rv} \\ &= \sqrt{v^T v} = ||v|| \end{aligned}$$

2-3 This follows from Problem 2-2 with $v = p_1 - p_2$.

2-9 Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SO(2).$$

From Cramer's rule and the fact that $A \in SO(3)$ we have

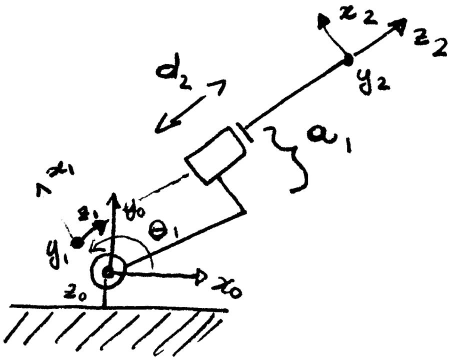
$$A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

which implies that $a = d$ and $b = -c$. Thus

$$A = \begin{bmatrix} a & -c \\ c & a \end{bmatrix}$$

with $\det A = 1 = a^2 + c^2$. Define $\theta = \tan^{-1}(c/a)$. Then $\cos\theta = a$ and $\sin\theta = c$.

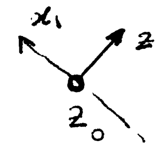
4. Two-link RP manipulator



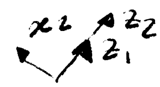
• z_0, y_1, y_2 coming out of the page

link	a	α	d	θ
ℓ_1	a_1	$\pi/2$	0	θ_1^*
ℓ_2	0	0	q_2^*	0

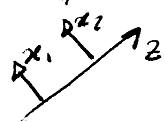
$\alpha_1 = \pi/2$: angle between z_0 and z_1 about x_1



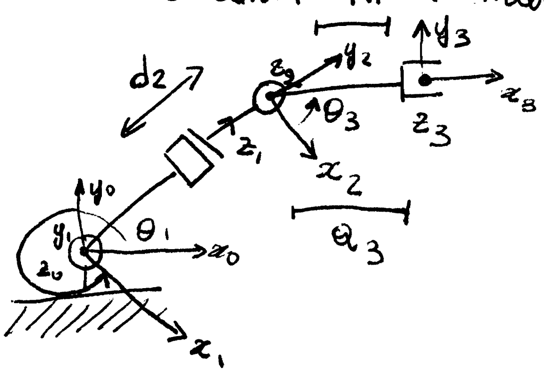
$\alpha_2 = 0$: " " z_1 " z_2 " x_2



$\theta_2 = 0$: " " x_1 " x_2 " z_1



Three-link RPR manipulator



∘ z_0, z_2, z_3 coming out of the page

∘ y_1 pointing into the page

$O_0 \equiv O_1$

[redacted]

link	a	α	d	θ
ℓ_1	0	$-\pi/2$	0	θ_1^*
ℓ_2	0	$\pi/2$	d_2^*	0
ℓ_3	a_3	0	0	θ_3^*