ECE 489 Robot Dynamics and Control

Spring 2007

Lab # 2: Simulation of Robot Dynamics

Due Date: Thursday, March 8, 2007

In this lab you will investigate the dynamic behavior of coupled rigid bodies (i.e., robots) via computer simulation using MATLAB.

Procedure:

1. Write a MATLAB program to simulate the dynamic equations of the planar elbow manipulator from the notes. You should create a file, `<filename.m>` containing the equations of motion in suitable form. See the description at the end of this assignment for hints on how to do this. The function `<filename.m>` should return the derivatives of the state variables, i.e.,

   ```
   xdot = [q1dot; q1doubledot; q2dot; q2doubledot];
   ```

   The MATLAB command

   ```
   [t,x] = ode45('<filename.m>',tspan, x0);
   ```

   can then be invoked to simulate the system, where `tspan` specifies the duration of the simulation, e.g.,

   ```
   tspan = [0 20];
   ```

   and `x0` is the vector of initial conditions. Refer to a MATLAB reference for additional details about how to simulate differential equations and plot the results.

   You should specify the dynamic parameters as `l1`, `m1`, etc., so that they can be easily changed later.

2. For this lab assignment, use the following values for the link parameters:

$$\begin{array}{llll} \ell_1 = 1.0 & \ell_{c1} = 0.5 & m_1 = 2 & I_1 = .50 \\ \ell_2 = 1.0 & \ell_{c2} = 0.5 & m_2 = 1 & I_2 = .25 \end{array} \tag{1}$$

3. Simulate the free response, i.e., $\tau_i = 0$, for forty seconds for the following initial conditions:

   - $q_1(0) = -1.5$; $q_2(0) = 0.1$; $\dot{q}_1(0) = 0$; $\dot{q}_2(0) = 0$.

   Note that the reference position $q_1 = 0$ is horizontal because of the way we have defined the joint variables so that the position $q_1 = -\pi/2$, $q_2 = 0$ corresponds to the robot hanging vertically downward.

4. Plot the link responses $q_1(t)$, $q_2(t)$. The motion should be nearly sinusoidal or you have made a mistake in your simulation.

5. Plot the phase space trajectories $\dot{q}_1(q_1)$, $\dot{q}_2(q_2)$.

6. Generate plots of the following generalized forces:
   - c) the Coriolis torques
   - d) the centrifugal torques
   - e) the gravitational torques
   - f) the inertial torques
7. Describe the relative magnitudes of each of these torques (Coriolis, centrifugal, gravitational, inertial). Which type seems the most important in terms of its effect on the response of the robot?
8. Simulate the free response for forty seconds for the following initial conditions:
   - $q_1(0) = 1.0$; $q_2(0) = 0$; $\dot{q}_1(0) = 0$; $\dot{q}_2(0) = 0$.
9. Plot the link responses and the phase space trajectories in this case. What is happening here? (Note: the answer to this question is extremely difficult. I only expect you to note the complexity of the response. This is an example of a so–called *chaotic* dynamical system.)
10. Generate plots of the Coriolis, centrifugal, gravitational, and inertial torques for this case. Compare the relative magnitudes of these torques with the torques generated in the first simulation run.
11. Modify the dynamic equations by adding friction terms $B_1\dot{q}_1$ to the first equation and $B_2\dot{q}_2$ to the second equation. For $B_1 = B_2 = 1$ repeat the first simulation run in 5) above. Plot the link responses and phase plane responses. Describe the qualitative behavior of the system in this case.

What to Turn In?

Turn in a listing of your MATLAB program and all requested plots. Make sure the plots are suitably labeled. Also answer all of the discussion questions above and submit the answers with your lab writeup.

How to Simulate a Second Order System

In order to simulate the nonlinear system

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{2}$$

in MATLAB, or most other packages, such as Mathematica, etc., you must first solve for the acceleration vector $\ddot{q}$ as

$$\ddot{q} = D(q)^{-1}\{\tau - C(q,\dot{q})\dot{q} - g(q)\} \tag{3}$$

For the 2–link planar RR–manipulator example, we have

$$\begin{aligned}
&d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1 \\
&d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2
\end{aligned}$$

For simplicity, define

$$\begin{aligned}
a_1 &= \tau_1 - c_{121}\dot{q}_1\dot{q}_2 - c_{211}\dot{q}_2\dot{q}_1 - c_{221}\dot{q}_2^2 - \phi_1 \\
a_2 &= \tau_2 - c_{112}\dot{q}_1^2 - \phi_2
\end{aligned}$$

Then the equations of motion may be succintly written as

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

This is just a system of two equations in two unknowns. Recall that the inverse of a 2×2 matrix

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

is just

$$\frac{1}{\Delta}\begin{bmatrix} d_{22} & -d_{12} \\ -d_{21} & d_{11} \end{bmatrix}$$

where $\Delta = d_{11}d_{22} - d_{21}d_{12}$ is the determinant of the matrix. Note that Δ is always non–zero because the robot inertia matrix is always positive definite.

Therefore the acceleration vector is given by

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} d_{22} & -d_{12} \\ -d_{21} & d_{11} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Multyplying this out gives,

$$\begin{aligned}
\ddot{q}_1 &= \frac{1}{\Delta}(d_{22}a_1 - d_{12}a_2) \\
\ddot{q}_2 &= \frac{1}{\Delta}(-d_{21}a_1 + d_{11}a_2)
\end{aligned}$$

which can easily be put into MATLAB syntax.