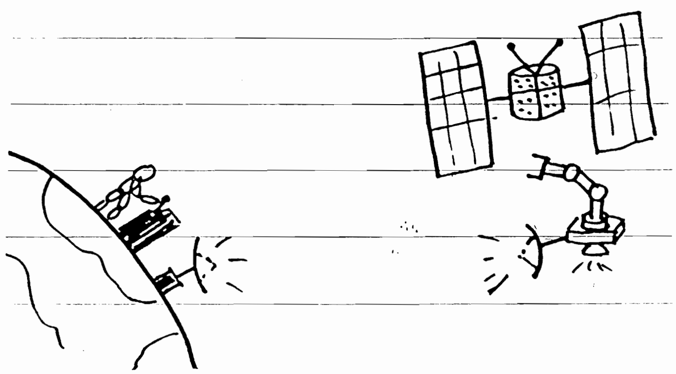
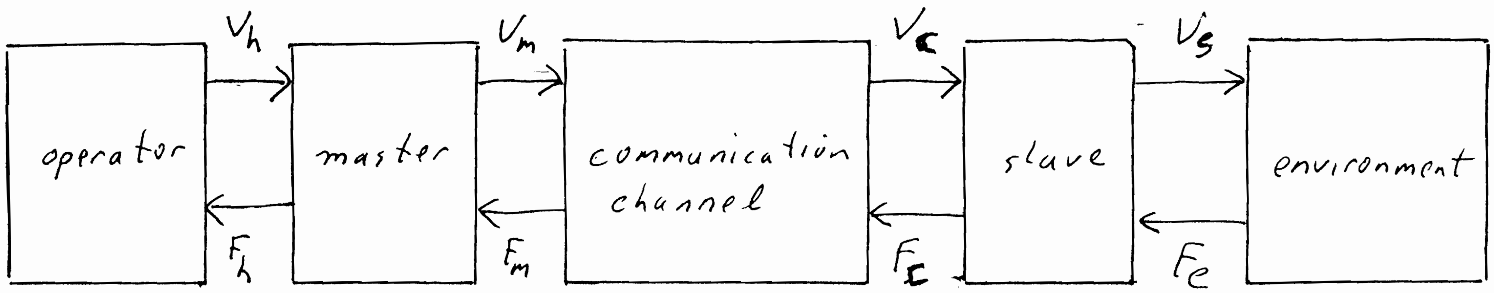
Introduction to Bilateral Teleoperation

A Teleoperation system consists of a master, which can be a robot, joystick, or other haptic device, and a slave, which is likewise a robot or other mechanism, connected via a communication medium, which can be a dedicated transmission line, a wireless or acoustic link, the Internet, etc.



A human operator manipulates the master to produce motion commands which are transmitted to the slave over the communication link. The slave interacts with the environment and communicates contact (force) information back to the master. Such a system is said to be bilateral and provide a sense of telepresence to the operator.



The major components of a teleoperator system are shown above. Velocity (motion) commands 'flow' from left to right while force signals return from right to left

We will consider mainly the dynamics of the master — communication — slave subsystems. A simple model of the master/slave systems is given by

$$M_m \dot{V}_m = \tau_m + F_h$$
$$M_s \dot{V}_s = \tau_s - F_e$$

where V_m, V_s are the master and slave velocities, τ_m, τ_s are the control inputs to the master and slave, and F_h, F_e are the forces from the human operator and the environment, respectively.

A simple model for the communication subsystem is a pure time delay. Thus,

$$V_e(t) = V_m(t-T)$$
$$F_m(t) = F_e(t-T)$$

where T (sec) is the communication delay. A simple control strategy is to choose

$$\tau_m = -B_m V_m - F_m$$
$$\tau_s = -B_s V_s + F_e$$

where

$$F_e = -B\left(V_e - V_s\right) + K_s \int_0^t \left(V_e(u) - V_s(u)\right) du$$

is called the coordinating torque, and is designed to cause the slave to track the master. For simplicity we write

$$F_s = -B\left(V_e - V_s\right) - K\left(X_e - X_s\right).$$

In case the time delay is negligible, we have

$$M_m \dot{V}_m + B_m V_m = F_h - F_m \quad ; \quad \text{and} \quad F_m = F_e$$
$$M_s \dot{V}_s + B_s V_s = F_e - F_e \qquad\qquad V_e = V_m$$

Suppose the operator moves the master to a constant position, i.e. a step change in master position. In the steady state, i.e. v_m, v_s, $\dot{v}_m$, $\dot{v}_s$ is equal to zero, we have

$$0 = F_h - K(x_e - x_s)$$

$$0 = K(x_e - x_s) - F_e$$

and so $$\boxed{K(x_e - x_s) = F_h = F_e}$$

This means that if $F_e = 0$ so that there is no contact force from the environment, the $x_e = x_s = x_m$, i.e. the slave tracts the master (up to a constant of integration).

Likewise if there is a non-zero contact force F_e this force is communicated to the operator, since $F_h = F_e$, ~~and~~ via a steady state error between master and slave.

In the case that the delay T is not negligible we have

$$V_c(t) = V_m(t-T) \; ; \; F_m(t) = F_c(t-T)$$

and the system equations become

$$M_m \dot{V}_m + B_m V_m = F_h - F_c(t-T)$$
$$M_s \dot{V}_s + B_s V_s = F_c(t) - F_e$$

with

$$F_c(t) = B\Big(V_m(t-T) - V_s(t)\Big) + K\Big(X_m(t-T) - X_s(t)\Big)$$

and the analysis of both the stability and tracking performance becomes much more difficult. We will have to introduce some additional machinery in order to analyze this system.

Notation and Background

Let $\mathbb{R}_+$ be the set of non-negative real numbers and $\mathbb{R}^n$ the usual n-dim'l Euclidean space, $\mathcal{L}_2^n(\mathbb{R}_+)$ the space of functions $f: \mathbb{R}_+ \to \mathbb{R}^n$ that are square-integrable with

$$\|f\|_2^2 = \int_0^\infty \|f(u)\|^2 du < \infty$$

A function $T : \mathcal{L}_2^n(\mathbb{R}_+) \to \mathcal{L}_2^n(\mathbb{R}_+)$ is called an <u>operator</u>. The induced norm of T is

$$\|T\| = \sup_{\|f\| \leq 1} \|Tf\|_2$$

<u>Example:</u> Let $h(t) : \mathbb{R}_+ \to \mathbb{R}$ be the impulse response function of a linear system. For $u \in \mathcal{L}_2(\mathbb{R}_+)$ we have $y(t) = \int_0^t h(\sigma - t) u(\sigma)\, d\sigma$ as the output of the system

$$u \longrightarrow \boxed{h} \longrightarrow y$$

Then $\|y\|_2 \leq \|h\| \cdot \|u\|_2$ so if h is a bounded operator, the norm of h defines a <u>gain</u> between input and output.

In the Laplace domain $Y(s) = H(s)\, \bar{U}(s)$ where $H(s) = \mathcal{L}\{h(t)\}$ is the transfer function. For a 1-port network

$$\boxed{Z} \begin{array}{l} \longleftarrow v \\ \longrightarrow F \end{array}$$

we have $F(t) = \int_0^t z(t-\sigma) v(\sigma) d\sigma$; $F(s) = Z(s) \bar{V}(s)$

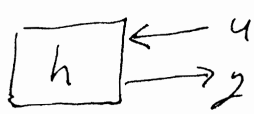
where $Z(s)$ is the impedance operator.

The 1-port is <u>passive</u> provided there exists a constant $\beta \geq 0$ such that

$$\int_0^t F^T(\sigma)\, v(\sigma)\, d\sigma \geq -\beta \quad \text{for all } t \geq 0$$

i.e. the integral is <u>lower bounded</u>.

$F^T v$ has units of power and $\int_0^t F^T(\sigma) v(\sigma)\, d\sigma$ is energy. A passive network therefore dissipates energy but does not create energy.

<u>Proposition:</u>  Let h represent a pure time delay, so that $y(t) = u(t-T)$. Then h is <u>not</u> a passive operator.

Proof: chose $u(t) = \sin t$, $T = \pi$. Then

$$\int_0^t y(\sigma) u(\sigma)\, d\sigma = \int_0^t \sin(\sigma - \pi) \sin(\sigma)\, d\sigma$$

$$= -\int_0^t \sin^2(\sigma)\, d\sigma \rightarrow -\infty \quad \text{as } t \rightarrow \infty,$$

and hence is not lower bounded.

Non-passivity of the time delay operator has a destabilizing effect on control systems.

Scattering Theory

Define scattering variables, or wave variables

$$s^+ = \frac{1}{\sqrt{2b}}(F + bv) \qquad \text{incident wave}$$

$$s^- = \frac{1}{\sqrt{2b}}(F - bv) \qquad \text{reflected wave}$$

The parameter b is introduced for scaling and has units of impedance.

The scattering operator S is define by

$$s^- = S s^+$$

and maps the "input wave" to the "output wave". It is easy to show that

$$S = (Z - bI)(Z + bI)^{-1}$$

where Z is the impedance of the 2-port.

Theorem: A 2-port network is passive iff $\|S'\| \le 1$.

Proof: If $\|S'\| \le 1$ then

$$\|F - bv\| / \|F + bv\| \le 1 \Rightarrow \|F + bv\| - \|F - bv\| \ge 0$$

writing this out gives

$$0 \le \int_0^t \left\{ (F+bv)^T (F+bv) - (F-bv)^T (F-bv) \right\} d\sigma$$

$$= 4b^2 \int_0^t F^T(\sigma) v(\sigma)\, d\sigma \Rightarrow \text{ the network}$$

is passive. Reversing the argument gives necessity.

We can write this in terms of the wave variables as follows:

with $\quad s^+ = \frac{1}{\sqrt{2b}}(F + bv); \quad s^- = \frac{1}{\sqrt{2b}}(F - bv)$

we have $\quad F = \sqrt{b/2}\,(s^+ + s^-); \quad v = \frac{1}{\sqrt{2b}}(s^+ - s^-)$

Then it follows that

$$\int_0^t F^T(\sigma) v(\sigma)\, d\sigma = \frac{1}{2} \int_0^t \left\{ s^{+T}(\sigma) s^+(\sigma) - s^{-T}(\sigma) s^-(\sigma) \right\} d\sigma$$

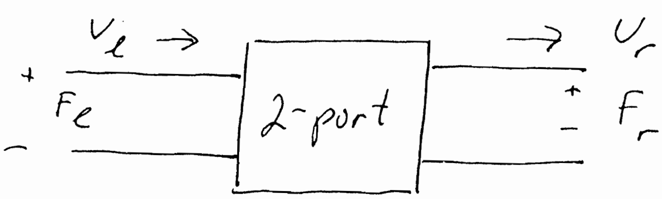
The network is therefore passive provided

$$\int_0^t s^{+T}(\sigma)\, s^{+}(\sigma)\, d\sigma \geq \int_0^t s^{-T}(\sigma)\, s^{-}(\sigma)\, d\sigma$$

which says, in effect, that the energy in the output wave is bounded by the energy in the input wave.

Two-port Networks

The above concept extend to more general n-port networks. We shall treat on 2-port networks here.



We define hybrid parameters, h_{ij} as

$$\begin{bmatrix} F_e \\ -V_e \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_r \\ F_r \end{bmatrix}$$

so that

$$h_{11} = \left.\frac{F_e}{V_e}\right|_{F_r=0} \; ; \quad h_{12} = \left.\frac{F_e}{F_r}\right|_{V_e=0}$$

$$h_{21} = \left.\frac{V_r}{V_e}\right|_{F_r=0} \; ; \quad h_{22} = \left.\frac{V_r}{F_r}\right|_{V_e=0}$$

As before we can define wave variables

$$s_e^+ = \frac{1}{\sqrt{2b}}\left(F_e + b v_e\right)$$

$$s_e^- = \frac{1}{\sqrt{2b}}\left(F_e - b v_e\right)$$

$$s_r^+ = \frac{1}{\sqrt{2b}}\left(F_r + b v_r\right)$$

$$s_r^- = \frac{1}{\sqrt{2b}}\left(F_r - b v_r\right)$$

so that

$$F_e = \sqrt{b/2}\left(s_e^+ + s_e^-\right)$$

$$v_e = \frac{1}{\sqrt{2b}}\left(s_e^+ - s_e^-\right) \quad ; \text{ etc.}$$

and we define the scattering operator by

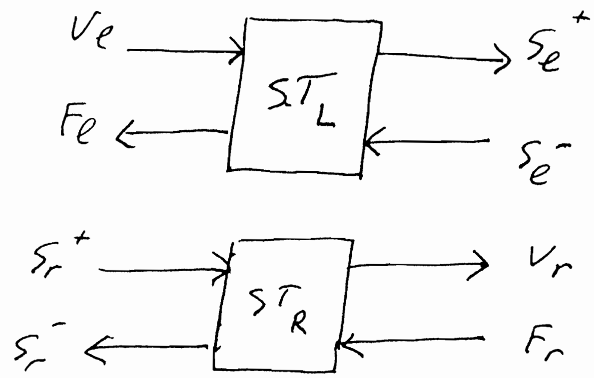
$$\begin{bmatrix} s_e^- \\ s_r^- \end{bmatrix} = S \begin{bmatrix} s_e^+ \\ s_r^+ \end{bmatrix}$$

and relate this to the hybrid parameters as

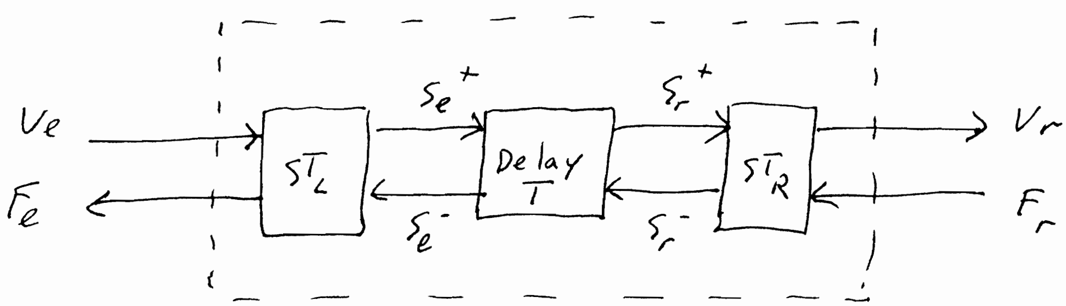
$$S = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\left(H - I\right)\left(H + I\right)^{-1}$$

and it follows that the 2-port network is passive iff $\|S\| \leq 1$.

Let us represent the scattering transformation as a 2-port:



Theorem: Consider the (cascade) network below



Then the network is passive for any delay T.

Proof: The proof is by direct calculation. Note that

$$s_r^+(t) = s_e^+(t-T)$$

$$s_e^-(t) = s_r^-(t-T)$$

Then

$$\int_0^t \{ F_e^T(\sigma) v_e(\sigma) - F_r^T(\sigma) v_r(\sigma) \} d\sigma$$

$$= \frac{1}{2} \int_0^t \Big\{ s_e^{+T}(\sigma)\, s_e^{+}(\sigma) - s_e^{-T}(\sigma)\, s_e^{-}(\sigma) - s_r^{+T}(\sigma)\, s_r^{+}(\sigma)$$
$$+ s_r^{-T}(\sigma)\, s_r^{-}(\sigma) \Big\}\, d\sigma$$

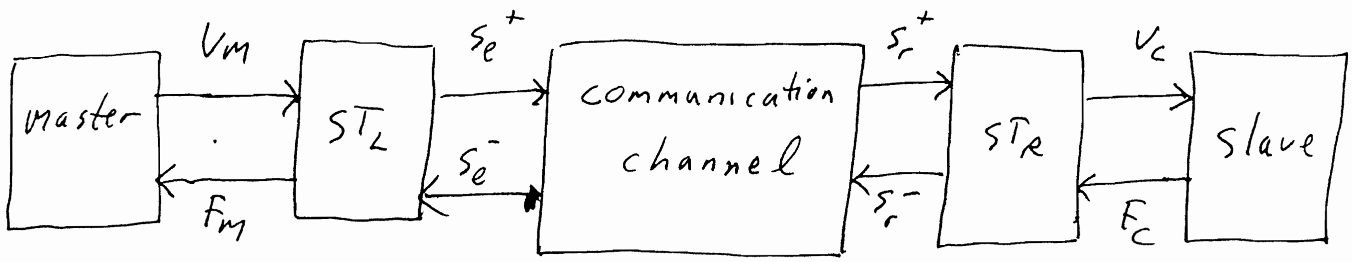
$$= \frac{1}{2} \int_0^t \Big\{ s_e^{+T}(\sigma)\, s_e^{+}(\sigma) - s_r^{-T}(\sigma - T)\, s_r^{-}(\sigma - T)$$
$$- s_e^{+T}(\sigma - T)\, s_e^{+}(\sigma - T) + s_r^{-T}(\sigma)\, s_r^{-}(\sigma) \Big\}\, d\sigma$$

$$= \frac{1}{2} \int_{t-T}^{t} \Big\{ s_e^{+T}(\sigma)\, s_e^{+}(\sigma) + s_r^{-T}(\sigma)\, s_r^{-}(\sigma) \Big\}\, d\sigma \geq 0$$

This latter integral represents the "energy" in the delay network over the interval $[t-T, t]$. Passivity means here that the energy dissipated "in transit" is non-negative.

We can use this result to define a control architecture for the master-slave teleoperator that preserves passivity in the presence of time delay.

The architecture:



Instead of transmitting the raw velocity and force signals between master and slave, we "encode" the velocity and force data into the wave variables and transmit the wave variables across the delay line.

<u>Proposition:</u> With the above communication architecture

$$F_m(t) = F_c(t-T) + b\left(V_m(t) - V_c(t-T)\right)$$

$$V_c(t) = V_m(t-T) + \frac{1}{b}\left(F_m(t-T) - F_c(t)\right)$$

and the entire system is passive assuming the human, master, slave, enviroment are passive.