

ECE 561: Detection and Estimation Theory
Spring 2009
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- 1)

Read the treatment of asymptotic robustness and robust detection on pages 250-263 of Levy, and provide a concise, one-or-two page summary of the most important results in the text.

- 2)

Under which conditions on the pdf's and loss function does there exist a natural ordering among the performances of MMSE, MAE, and MAP estimators (in class, we showed what the conditions are for all three to be identical. This questions asks you to establish when can you claim one to be strictly larger than the other two, for all possible pairs and one triple).

- 3)

Consider the modified LLS problem, in which you seek an estimator of \mathbf{X} of the form $\hat{\mathbf{X}} = A \mathbf{Y} + \mathbf{b}$, but with A required to be upper or lower triangular. As an example, you could also consider another form of a LLS estimator with structural constraints on A . How would you solve this problem? When does the LLS estimator allow for A to be triangular?

- 4)

Prove the following statements about the Schur complement D/M of a matrix D with respect to the block matrix M , where M consists of blocks A, B, C, D , respectively:

a) $\det(M) = \det(D) \times \det(D/M)$.

b) The *inertia* of a Hermitian matrix M is an ordered triple, $I_M = (m_+, m_-, m_0)$, where m_+ , m_- , and m_0 denote the number of positive, negative, and zero-valued eigenvalues of M . Show that

$$I_M = I_D + I_{D/M}.$$

c) Can you explain in an intuitive way how the Schur complement arises in the expressions for the LLSE error-covariance matrix?

Assume that, as in class, D is a non-singular matrix.

- 5)
 - a) In class, we showed that there exists a fundamental relationship between the variance of an estimator, and the Fisher information over the pdf of the observables (the Cramer-Rao inequality). Are there natural extensions for bounding the third order moment of the estimation error? What kind of properties would the "information measure" have to have to make this plausible?
 - b) Find the Fisher information contained in n independent Bernoulli trials.
 - c) Write a short report on the so-called Jeffreys prior distribution, related to Fisher information. Do you see where this prior could be used in our study of estimation theory?