ECE 563 Homework # 4 Due: November 10, 2009

1. Write a program to compute the capacity of channels with the following probability transmission matrices

a)

$$\boldsymbol{Q} = \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix}$$

b)

$$\boldsymbol{Q} = \begin{bmatrix} .8 & .1 & .1 \\ .3 & .3 & .4 \\ .1 & .1 & .8 \end{bmatrix}$$

2. Prove the useful rule

$$\prod_{\ell=1}^{n} \left(\sum_{j_\ell} A_{j_\ell} \right) = \sum_{j_1} \sum_{j_2} \cdots \sum_{j_n} \left(\prod_{\ell=1}^{n} A_{j_\ell} \right),$$

where $(A_{j_1}, \ldots, A_{j_n})$ is a vector whose range of values in the ℓth component is indexed by j_ℓ. (This is a general form of the identity $(a+b)(c+d) = ac + ad + bc + bd$.)

3. Prove that

$$I((X,Y);(U,V,W)) = I((X,Y):U|V,W) + I((X,Y);V|W) + I((X,Y);W).$$

4. Let U, V, X, Y, and Z be random variables. Do the conditions

$$I(V;Y,Z|U,X) = 0$$

and

$$I(X;U,Z|V,Y) = 0$$

imply that

$$I(Z;X,V|U,Y) = 0?$$

5. Suppose that $I(X;Y) = 0$. Does this imply that $I(X;Z) = I(X;Z|Y)$?

6. Consider a source that generates a sequence of binary symbols. Successive symbols generated by the source are independent, and symbols zero and one each occur with probability $\frac{1}{2}$. You are to compress this output data. The reproducing alphabet is $\{0, 1, e\}$ where the symbol e denotes an *erasure*. The distortion matrix is

$$\rho = \begin{bmatrix} 0 & 1 & 0.25 \\ 1 & 0 & 0.25 \end{bmatrix}.$$

That is, an error is judged to be four times as severe as an erasure. By what factor can the data be compressed so that the average distortion is no worse than D?

7. Suppose that a binary equiprobable source is to be compressed. The distortion matrix is

$$\rho = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}.$$

That is, it is twice as serious to reproduce a zero by a one as it is to reproduce a one by a zero. Find $R(D)$.