ECE 467 COMMUNICATION NETWORK ANALYSIS **SPRING 2002**
Exam 1 **Tuesday, February 26**
NAME: ______________________________

You have 70 minutes to complete the exam. You may use one sheet of notes (two-sided). No other books or notes are allowed. Show your work for full credit. There are four problems weighted as indicated for a total of 30 points (10 points per page).

1. Consider an M/M/2/3 queueing system (2 servers, room for 3 customers in the system) with Poisson arrival process of rate λ and exponential service times with parameter μ. Customers arriving to find the system full are dropped from the system.
(a) (2 pts) Sketch the rate transition diagram.

(b) (2 pts) Sketch the Q matrix.

(c) (2 pts) Find the equilibrium distribution.

(d) (4 pts) What is the long term arrival rate of customers that never see another customer in the system? (Those are customers that arrive to find the system empty and depart before any other customers arrive.)

2. Consider a queueing system with an unlimited waiting room and two servers. The arrivals form a Poisson process of rate λ. Once a customer enters a server it remains in the server until service is completed, and then it exits the system. The service time distribution for server a is exponential with parameter μ_a and for server b is exponential with parameter μ_b. Server a is faster: $\mu_a > \mu_b$, so if a customer arrives to an empty system, the customer goes to server a. If there are two or more customers in the system, then both servers are busy.

(a) (2 pts.) Explain why the number of customers in the system is not a Markov process.

(b) (4 pts.) Sketch a rate transition diagram for a Markov process that describes the system, and indicate the meaning of each state.

(c) (2 pts.) Under what condition on λ, μ_a, and μ_b is the Markov process positive recurrent?

(d) (2 pts.) Assuming the Markov process is positive recurrent, let f_a denote the fraction of customers served by server a. Express f_a in terms of λ, μ_a, and μ_b and the equilibrium probabilities for a *finite* number of states.

3. **(4 pts.)** **Consider a single server queueing system such that the service times are independent and identically distributed with the same distribution as a random varible X. The arrival process is a modulated Poisson process. When the system is not empty, new arrivals occur according to a Poisson process of rate λ. But if the system is empty, arrivals occur according to a Poisson process of rate 2λ. Express the mean waiting time of a typical customer in terms of λ and the distribution of X. Also, state the condition under which the mean waiting time is finite. (Hint: This is an easy problem if you have a good way to think about it.)**

4. **(6 pts) Consider a discrete-time, time-homogeneous Markov process X with a countably infinite state space. Suppose the process is irreducible. Prove that if one state is recurrent, then every state is recurrent.**