ECE 467 COMMUNICATION NETWORK ANALYSIS **SPRING 2002**
Exam 2 **Wednesday, April 10**
NAME: ______________________________

You have 70 minutes to complete the exam. You may use two sheets of notes (two-sided). No other books or notes are allowed. Show your work for full credit. There are four problems weighted as indicated for a total of 30 points.

1. (a) (3 pts) Consider the following arrival process. Packets arrive according to a Poisson process with rate $\lambda > 0$ and each packet is 10^4 bits long. Compute the effective bandwidth $\alpha(s, t)$ (in bits per second) for $s, t > 0$.

(b) (3 pts) Suppose A is a random arrival process that satisfies deterministic constraints: specifically it is (σ, ρ)-upper constrained for some positive parameters σ and ρ. Compute the tightest possible upper bound on the effective bandwidth function $\alpha(s, t)$ of A for $s, t > 0$.

2. (4 pts) Consider the number of customers $N = (N(t) : t \in R)$ in a stable, stationary M/GI/1 queueing system with first-in first-out (FIFO) service order.

Is N necessarily time reversible? Justify your answer.

3. (*Two buffered stations with Bernoulli arrivals using ALOHA*) Consider two stations, 1 and 2, operating in discrete time. If station i is not empty at the beginning of a slot, it transmits a packet with probability q_i, independently of the state of the other station. If only one station transmits a packet in the slot, the packet exits the system. Otherwise no packet departs either station. After the transmission attempt, if any, a new packet is added to each station i with probability p_i. The system can be modeled as a Markov process with an infinite two-dimensional state space. Here state (n_1, n_2) means station i has n_i packets at the beginning of a slot for $i = 1$ and $i = 2$. The drift for any state (n_1, n_2) is a two dimensional vector, and together the drifts for all states form a drift vector field.

(a) (5 pts) Sketch and clearly indicate the numerical values of the drift field for the case $p_1 = p_2 = 0.1$ and $q_1 = q_2 = 0.5$. State whether the system is stable, and indicate the level sets of a Lyapunov function that can be used to prove stability or instability.

(b) (5 pts) Repeat part (a) for $p_1 = 0.3,\ p_2 = 0.1$ **and** $q_1 = q_2 = 0.5$**.**

4. Let Z_+ denote the nonnegative integers. In the following, functions f and g are nonnegative nondecreasing functions from Z_+ to Z_+. Write $f \wedge g$ for the minimum of f and g, so $(f \wedge g)(t) = \min\{f(t), g(t)\}$ for $t \geq 0$. The following two facts may be used without proof:
(1) If $f(0) = 0$, then $f \star g \leq g$.
(2) If f is sublinear and f(0)=0, then $f \star f = f$.
(a) (4 pts) Prove that if $f(0) = g(0) = 0$ and if $f \wedge g$ is sublinear, then $f \wedge g = f \star g$. (Hint: It suffices to prove that $f \wedge g \leq f \star g$ and $f \wedge g \geq f \star g$.)

(b) (3 pts) Consider two token bucket filters in series. The first with parameters $(\sigma_1, \rho_1) = (20, 2)$ and the second with parameters $(\sigma_2, \rho_2) = (10, 4)$. This series forms a maximal f-regulator for a certain function f. Carefully sketch f and label the sketch.

(c) (3 pts) For the series of token bucket filters described in part (b), what is the maximum delay of a packet in the second filter?