ECE 467 COMMUNICATION NETWORK ANALYSIS — **Spring 2002**
Final Exam — **Monday, May 6**
NAME: __________________

You have three hours to complete the exam. You may use three sheets of notes (two-sided). No other books or notes are allowed. Show your work for full credit. Each part of each problem receives equal weight. There are 6 problems on 8 pages.

1. (Joint routing/fair allocation) Consider a network with three links, each with capacity one, and four flows. For $1 \leq i \leq 3$ flow i has one route, consisting of link i alone. Flow 4 can be split between two routes: one consisting of links 1 and 2, and the other consisting of link 3 alone. Let r_i denote the total rate assigned to flow i.

(a) Find the max-min fair allocation vector (r_1, r_2, r_3, r_4). Also, indicate how flow 4 is allocated between the two routes.

(b) Find the proportionally fair allocation vector (r_1, r_2, r_3, r_4) (assuming the flows have equal weights). Also, indicate how flow four is allocated between the two routes, and indicate the price for each link.

2. **Consider a** $GI/M/2$ **system with first-come, first-served order of service. Let** T **deonte a typical interarrival time, and let** μ **denote the service rate for each of the two servers. Suppose that the mean arrival rate,** $1/E[T]$**, is less than** 2μ**, the service rate when there are two or more customers in the system.**

(a) Identify a discrete-time discrete-state Markov chain embedded in the system. (Indicate the state-space, the significance of each state, and the one-step transition probabilities.)

(b) How does the mean delay for this system compare to the mean delay in a $GI/M/1$ **system with the same arrival process and with service rate** 2μ**? Justify your answer for full credit.**

3. Consider a queue with customers of two classes. Customers of class i arrive according to a Poisson process of rate λ_1 for $i = 1, 2$. The service rate μ is shared between the two classes in a controlled way. Class 1 receives service rate $u\mu$ and the second class receives service rate $(1-u)\mu$, where u is a control variable with $0 \leq u \leq 1$. The insantaneous cost function is $g(x_1, x_2) = c_1x_1 + c_2x_2$ where, for $i = 1$ or $i = 2$, x_i is the number of customers in station i and c_i is a positive constant.
(a) Suppose that a Poisson process of rate $\gamma = \lambda_1 + \lambda_2 + \mu$ is used to signal the occurence of events. Describe the corresponding one-step transition probability matrix $P(u)$ for the system observed at event times.

(b) Give the backwards dynamic programming equations for the the optimal cost-to-go functions V_n for $n \geq 1$. (Assume a discount factor β as in class.) Be as explicit as you can.

(c) Explicitly describe how the optimal control can be deduced from V_n.

(d) Outline a proof of optimality of the "μc rule", which is the strategy of serving the station with the larger value of $\mu_i c_i$ whenever that station is not empty.

4. Indicate whether each of the following statements is true or false. *If false, give a counter example. If true, give a brief explanation.*

(a) Consider a birth-death Markov process with arrival rates $(\lambda_k : k \geq 0)$ **and departure rates** $(\mu_k : k \geq 1)$**. If the sequence** $(\lambda_k/k : k \geq 1)$ **is bounded above by a finite constant, then the birth-death process is nonexplosive.**

T or F?

(b) If a token bucket filter with parameters (σ_1, ρ_1) **is followed by a token bucket filter with parameters** (σ_2, ρ_2)**, then there is a finite constant** B **depending only on** $(\sigma_1, \rho_1, \sigma_2, \rho_2)$ **such that for any cummulative arrival waveform, the number of packets queued in the second filter is less than or equal to** B**.**

T or F?

(c) The minimum capacity cut in a flow network $(s, t, \mathcal{N}, \mathcal{L}, C)$ is unique. (Here s and t are the source and terminal nodes, $\mathcal{N}$ is the set of nodes, $\mathcal{L}$ is the set of directed links, and C is the vector of link capacities.)

T or F?

(d) For any single-server queue modeled by a positive-recurrent birth-death process with finite mean arrival rate, the distribution of the number of customers in the system seen by arrivals is equal to the equilibrium distribution (i.e. distribution seen over long time intervals) of the number of customers in the system.

T or F?

5. Consider the communication network with 24 links indicated in Figure 1. Each undirected link in the figure represents two directed links, one in each

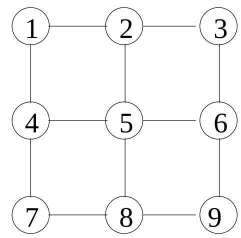

Figure 1: Mesh network with one source and one destination

direction. There are four origin destination pairs: $W = \{(1,9),(3,7),(9,1),(7,3)\}$**, each with demand** a**. The initial routing is deterministic, being concentrated on the four paths** $(1,2,5,6,9)$**,**$(3,6,5,8,7)$**,**$(9,8,5,4,1)$**, and** $(7,4,5,2,3)$**. Suppose that the cost associated with any one of the 24 links is given by** $D(F) = F^2/2$**, where** F **is the total flow on the link, measured in units of traffic per second.**
(a) What is the cost associated with the initial routing?

(b) Describe the flow (for all four destinations) after one iteration of the flow deviation algorithm. Assume that, for any given origin-destination pair, any route from the origin to the destinationcan be used.

(c) Is the resulting routing optimal? If so, prove it. If not, give an optimal flow.

6. **(Basic tree algorithm with feedback errors) Consider the basic unmodified tree random access algorithm with the following modification. Let $0 \leq \epsilon < 1$. If in a slot there is exactly one transmission, then with probability ϵ all stations get feedback "e" (so they mistakenly think there was a collision), and with probability $1-\epsilon$ all stations get feedback "1"(so they know there was a successful transmission). In the other cases (idle slot or collision slot) the feedback is error free. Let V_n denote the expected number of slots needed for the algorithm to finish, given that it is applied to a batch of n stations.**

(a) Find V_0, V_1, and V_2 explicitly in terms of ϵ.

(b) Derive a recursion for the sequence $(V_n : n \geq 0)$.

Best wishes for the summer!