University of Illinois at Urbana-Champaign

ECE 467 COMMUNICATION NETWORK ANALYSIS

Fall 2006
Exam I

Monday, October 16, 2006

Name: ____

- You have 75 minutes for this exam. The exam is closed book and closed note, except you may consult both sides of one $8.5'' \times 11''$ sheet of notes in ten point font size or larger, or equivalent handwriting size.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.
- If you feel that a question is ambiguous, give your reasons and state your assumptions.

Score:

1. __________ (10 pts.)

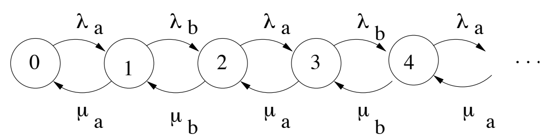
2. __________ (10 pts.)

3. __________ (5 pts.)

4. __________ (5 pts.)

Total: ________(30 pts.)

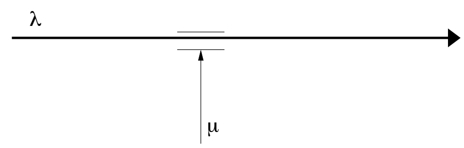
1. Consider a single server queueing system in which the number in the system is modeled as a continuous time birth-death process with the transition rate diagram shown, where $\lambda_a, \lambda_b, \mu_a$, and μ_b are strictly positive constants.



(a) Under what additional assumptions on these four parameters is the process positive recurrent?

(b) Assuming the system is positive recurrent, under what conditions on $\lambda_a, \lambda_b, \mu_a$, and μ_b is it true that the distribution of the number in the system at the time of a typical arrival is the same as the equilibrium distribution of the number in the system?

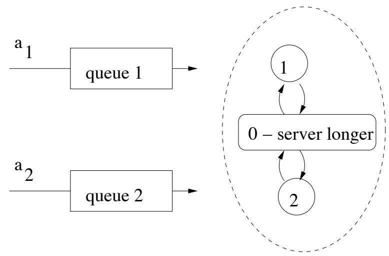
2. Suppose that a regulated communication link resets at a sequence of times forming a Poisson process with rate μ. Packets are offered to the link according to a Poisson process with rate λ. Suppose the link shuts down after three packets pass in the absence of resets. Once the link is shut down, additional offered packets are dropped, until the link is reset again, at which time the process begins anew.



(a) Sketch a transition rate diagram for a finite state Markov process describing the system state.

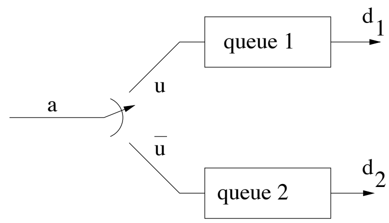
(b) Express the dropping probability (same as the long term fraction of packets dropped) in terms of λ and μ.

3. Consider two queues, queue 1 and queue 2, such that in each time slot, queue i receives a new packet with probability a_i, where $0 < a_1 < 1$ and $0 < a_2 < 1$. Suppose the server is described by a three state Markov process, as shown.



If the server process is in state i for $i \in \{1, 2\}$ at the beginning of a slot, then a potential service is given to station i. If the server process is in state 0 at the beginning of a slot, then a potential service is given to the longer queue (with ties broken in favor of queue 1). Then during the slot, the server state jumps with the probabilities indicated. (Note that a packet can arrive and depart in one time slot.) For what values of a_1 and a_2 is the process stable? Briefly explain your answer (but rigorous proof is not required).

4. Consider the system of Example 1.b in the notes (a discrete time model, using the route to shorter policy, with ties broken in favor of queue 1, so $u = I_{\{x_1 \le x_2\}}$):



Assume $a = 0.7$ and $d_1 = d_2 = 0.4$. The system is positive recurrent. Explain why the function $V(x) = x_1 + x_2$ does *not* satisfy the Foster-Lyapunov stability criteria for positive recurrence, for any choice of the constant b and the finite set C.