University of Illinois at Urbana-Champaign

ECE 467 COMMUNICATION NETWORK ANALYSIS

Fall 2006
Exam II

Monday, November 13, 2006

Name: _____

- You have 75 minutes for this exam. The exam is closed book and closed note, except you may consult both sides of one $8.5'' \times 11''$ sheet of notes in ten point font size or larger, or equivalent handwriting size.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. _____ (5 pts.)

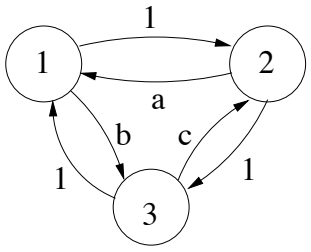
2. _____ (5 pts.)

3. _____ (10 pts.)

4. _____ (10 pts.)

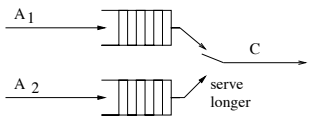
Total: _____(30 pts.)

1. (5 points) Consider the continuous-time Markov process with the transition rate diagram shown. Identify all possible values of $(a, b, c) \in \mathbb{R}^3_+$ which make the process reversible.



2. (5 points=3 points for correct answer + 2 points for proof) Suppose X is a random variable representing the rate of a connection sharing a bufferless link. Suppose the only information known about the probability distribution of X is that $E[X] = 1$ and $P[0 \leq X \leq 2] = 1$. For a given $s > 0$ fixed, what specific choice of distribution of X meeting these constraints maximizes the equivalent bandwidth $\alpha(s)$, where $\alpha(s) = \frac{\ln(E[e^{sX}])}{s}$?

3. Suppose two queues are served by a constant rate server with service rate C, which serves the longer queue. Specifically, suppose a discrete-time model is used, and after new customers arrive in a given slot, there are C potential services available, which are allocated one at a time to the longer queue. For example, if after the arrivals in a slot there are 9 customers in the first queue, and 7 in the second, and if $C = 4$, then 3 customers are served from the first queue and 1 from the second. Suppose the arrival process A_i to queue i is (σ_i, ρ_i)-upper constrained, for each i, where $\sigma_1, \sigma_2, \rho_1, \rho_2$, and C are strictly positive integers such that $\rho_1 + \rho_2 \leq C$.

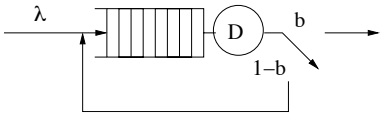


(a) (3 points) Find a bound on the maximum number of customers in the system, carried over from one slot to the next, which is valid for any order of service within each substream.

(b) (3 points) Find a bound on the maximum delay which is valid for any order of service within each substream. (The bound should be finite if $\rho_1 + \rho_2 < C$.)

(c) (4 points) Suppose in addition that customers within each stream are served in FIFO order. Find an upper bound for the delay of customers in stream 1 which is finite, even if $\rho_1 + \rho_2 = C$. Explain your reasoning.

4. Consider a queue with feedback as shown, where $\lambda, D > 0$ and $0 \leq b \leq 1$. New arrivals occur according to a Poisson process of rate λ. The service time of a customer for each visit to the queue is the constant D. Upon a service completion, the customer is returned to the queue with probability $1 - b$.



(a)(2 points) Under what condition is the system stable? Justify your answer.

(b)(5 points) Suppose the service order is FIFO, except that a returning customer is able to bypass all other customers and begin a new service immediately. Denote this by PR, for priority to returning, service order. Express the mean total system time of a customer, from the time it arrives until the time it leaves the server for the last time, in terms of λ, b, and D. (Hint: A geometrically distributed random variable with parameter p has first moment $\frac{1}{p}$ and second moment $\frac{2-p}{p^2}$.)

(c)(3 points) If instead the service within the queue were true FIFO, so that returning customers go to the end of the line, would the mean total time in the system be larger, equal, or smaller than for PR service order? Would the variance of the total time in the system be larger, equal, or smaller than for PR service order?